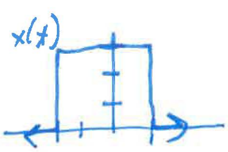
Name Answers

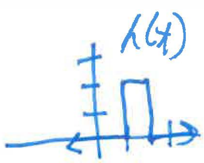
EE3032 – Dr. Durant – Quiz 4
Fall 2019, Week 4

Recall that the convolution integral is $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ and that convolution is commutative.

1. (1 point) Sketch x(t) = 3u(t+2) – 3u(t-1) and h(t) = 2u(t-1) – 2u(t-2)
2. (2 points) What is the range where the following functions are non-zero?
   a. x(t)
   b. h(t)
   c. y(t); solve this using the width property
3. (2 points) What is the area of the following functions? Show the key calculation steps, but it is not necessary to formally evaluate the area integrals.
   a. x(t)
   b. h(t)
   c. y(t); solve this using the area property
4. (5 points) Calculate y(t) using graphical convolution by following these steps:
   a. Fold h(t) and sketch the result, h(-t). This follows the form of convolution given above (not the commutative form).
   b. Overlay the folded h(t) on x(t) and calculate the area of the product. This is y(0).
   c. Shift the folded h(t) 2 seconds to the right and overlay it on a graph of x(t).
   d. Calculate the area of the product. This is y(2).
   e. Complete the graph of y(t). Hint: consider how the area increases and decreases as the folded h(t) shifts to the right.

(1)





(2)(a) −2 → 1

Accepted: width=3, but edges should be given

(b) 1 → 2

(c) Add edges per width property

(−2+1) → (1+2)

−1 → 3

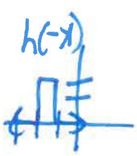
(3) (a) $A_x = W \cdot H = 3 \cdot 3 = 9$

(b) $A_h = 1 \cdot 2 = 2$

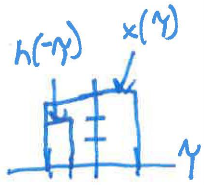
(c) Multiply areas per area property

$A_y = A_x A_h = 9 \cdot 2 = 18$

(4) (a)



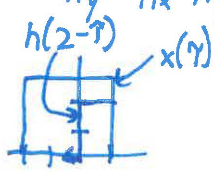
(b)



Product

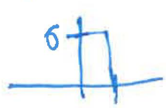
$y(0) = 1 \cdot 6 = 6$

(c)



Product

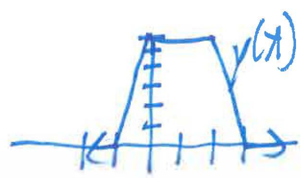
(d)



$y(2) = 1 \cdot 6 = 6$

(e) We see that t=0, 2 are the edges of the maximum area region (6).

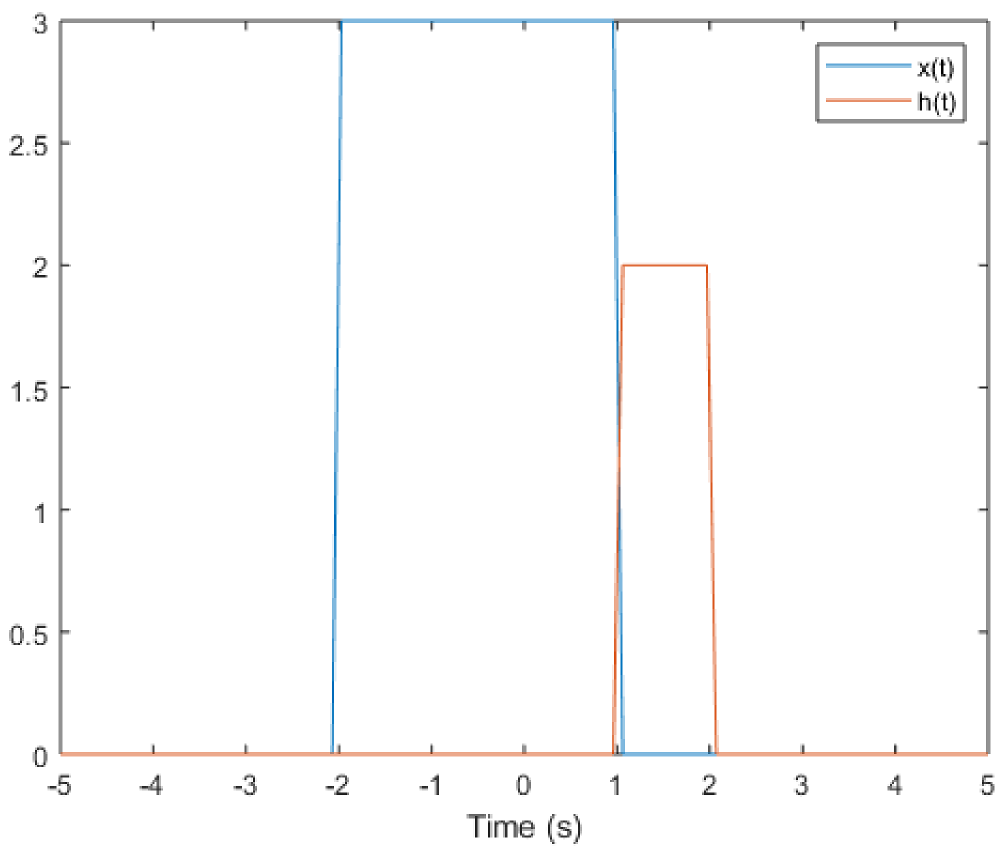
(2c) Tells us area is 0 outside of [−1,3].



EE3032: Dr. Durant's solution to Quiz 4, 10/7/2019

Problem 1: Plot the given functions

```
t = linspace(-5, 5);
x = 3*(t>-2) - 3*(t>1);
h = 2*(t> 1) - 2*(t>2);
figure, plot(t,x, t,h), legend('x(t)', 'h(t)'), xlabel('Time (s)')
```



Problem 2: Area of support, including for y(t) using width property

```
xt = [-2 1]; % put support limits in 2-element vector
ht = [1 2];
yt = xt+ht; % Convolution width property
fprintf('y(t) is active on interval [%g,%g] and has a width of %g.\n',...
    yt(1), yt(2), diff(yt))
```

```
y(t) is active on interval [-1,3] and has a width of 4.
```

Problem 3: Areas of each function, including for y(t) using area property. On the quiz, you'd multiply the width and the height of the rectangles. This method confirms the quiz ansers.

```
dt = diff(t(1:2)); % time step
Ax = dt*sum(x); % Approximate integral with Riemann sum
Ah = dt*sum(h);
```

```
Ay = Ax * Ah; % Convolution area property, dt*sum(y) also works
fprintf('Areas: x(t): %g; h(t): %g; y(t): %g\n', Ax, Ah, Ay)
```

```
Areas: x(t): 9.09091; h(t): 2.0202; y(t): 18.3655
```

Problem 4: Graphical convolution. See the handwritten solution for details. Here we just show how MATLAB can calculate the final answer for us, which we compare to what we calculated for the quiz.

```
y = dt*conv(x, h);
ty = linspace(2*t(1), 2*t(end), 2*length(t)-1); % convolution width property
figure
plot(t,x, t,h, ty,y), legend('x(t)', 'h(t)', 'y(t)'), xlabel('Time (s)')
xlim([t(1) t(end)]) % crop out the part of the expanded ty that doesn't overlap t
```