

EE3032 - Dr. Durant - Quiz 4
Fall 2017, Week 4

1. (3 points) Prove whether the following system is linear: $y(t) = 2x(t) - 3x(t-1)$
2. (2 points) Prove whether the following system is linear: $y(t) = \sin(x(t))$
3. (3 points) Prove whether the following system is time-invariant: $y(t) = \ln(t) x(t)$
4. (2 points) Prove whether the following system is BIBO stable: $y(t) = \ln(t) x(t)$

① Let $x(t) = x_1(t) + x_2(t)$

$$x_1(t): y_1(t) = 2x_1(t) - 3x_1(t-1)$$

$$x_2(t): y_2(t) = 2x_2(t) - 3x_2(t-1)$$

$$y_1(t) + y_2(t) = 2x_1(t) - 3x_1(t-1) + 2x_2(t) - 3x_2(t-1) = 2(x_1(t) + x_2(t)) - 3(x_1(t-1) + x_2(t-1))$$

$$= 2x(t) - 3x(t-1) = y(t) \quad \therefore \boxed{\text{Linear}}$$

② $y_1(t) + y_2(t) \stackrel{?}{=} y(t)$

$$\sin(x_1(t)) + \sin(x_2(t)) \stackrel{?}{=} \sin(x_1(t) + x_2(t))$$

Not true, would require $\sin(a) + \sin(b) = \sin(a+b)$!

$\therefore \boxed{\text{non-linear}}$

③ System, then delay: $y(t) = \ln(t) x(t) \rightarrow y(t-d) = \ln(t-d) x(t-d)$

Delay, then system: Input $x(t-d) \rightarrow y^+(t) = \ln(t) x(t-d)$

not equal

$\therefore \boxed{\text{not time-invariant}}$

④ Let $|x(t)| \leq M \quad \forall t$

$$y(t) \leq \ln(t) M$$

\uparrow
grows w/o bound as $t \rightarrow \infty$

$\therefore \boxed{\text{not BIBO stable}}$