HW 7 Solution - EE 3032 - Dr. Durant, - Fall, 2017

Problem 8.4, page 529

$x(t) = \delta(t+1) + \delta(t-1)$

(a) $X(\Omega) = e^{j\Omega} + e^{-j\Omega} = 2\cos(\Omega)$

Not bandlimited. $X(\Omega) \neq 0$ for Ω arbitrarily large

(b) $H(\Omega) = u(\Omega+1) - u(\Omega-1)$

$$Y(\Omega) = \begin{cases} 2\cos(\Omega), & |\Omega| < 1 \\ 0, & \text{otherwise} \end{cases}$$

$y(t)$ is bandlimited with a max. freq. of $\Omega = 1$ rad/sec.

(c) $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\cos(\Omega)[u(\Omega+1) - u(\Omega-1)] e^{j\Omega t} d\Omega$

$= \frac{1}{\pi}\int_{-1}^{1} \cos(\Omega) e^{j\Omega t} d\Omega$

~~$= \frac{1}{2\pi}\int_{-1}^{1} (e^{j\Omega} + e^{-j\Omega}) e^{j\Omega t} d\Omega$~~

$= \frac{1}{\pi}\int_{-1}^{1} \cos(\Omega)(\cos(\Omega t) + j\sin(\Omega t)) d\Omega$

$= \frac{1}{\pi}\int_{-1}^{1} \cos(\Omega)\cos(\Omega t) d\Omega$ [using $(\int_{-\alpha}^{\alpha} E \cdot O) = \emptyset$]

$= \frac{2}{\pi}\int_{0}^{1} \cos(\Omega)\cos(\Omega t) d\Omega$

$= \frac{1}{\pi}\int_{0}^{1} \cos(\Omega - \Omega t) + \cos(\Omega + \Omega t) d\Omega$ [using $2\cos\theta\cos p = \cos(\theta - p) + \cos(\theta + p)$]

$= \frac{1}{\pi}\int_{0}^{1} \cos(\Omega(1-t)) + \cos(\Omega(1+t)) d\Omega$

$= \frac{1}{\pi}\left[\frac{1}{1-t}\sin(\Omega(1-t)) + \frac{1}{1+t}\sin(\Omega(1+t))\right]_{\Omega=0}^{1}$

$= \frac{1}{\pi}\left[\frac{\sin(1-t)}{1-t} + \frac{\sin(1+t)}{1+t} - 0 - 0\right]$

$= \frac{1}{\pi}(\mathrm{sinc}(1-t) + \mathrm{sinc}(1+t))$

$= \frac{1}{\pi}(\mathrm{sinc}(t+1) + \mathrm{sinc}(t-1))$ [using sinc is even]

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>> hw7
Energy of y(t) is 0.900415 using the time-domain Riemann approach.
Error of finding energy using Parseval (power spectrum) is 3.05323%.
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% EE3032 - Problem 8.4, page 529 - Dr. Durant
% HW-7, Due Thursday of week 10

% The posted solution shows that
% y(t) = (sinc(t+1)+sinc(t-1))/pi

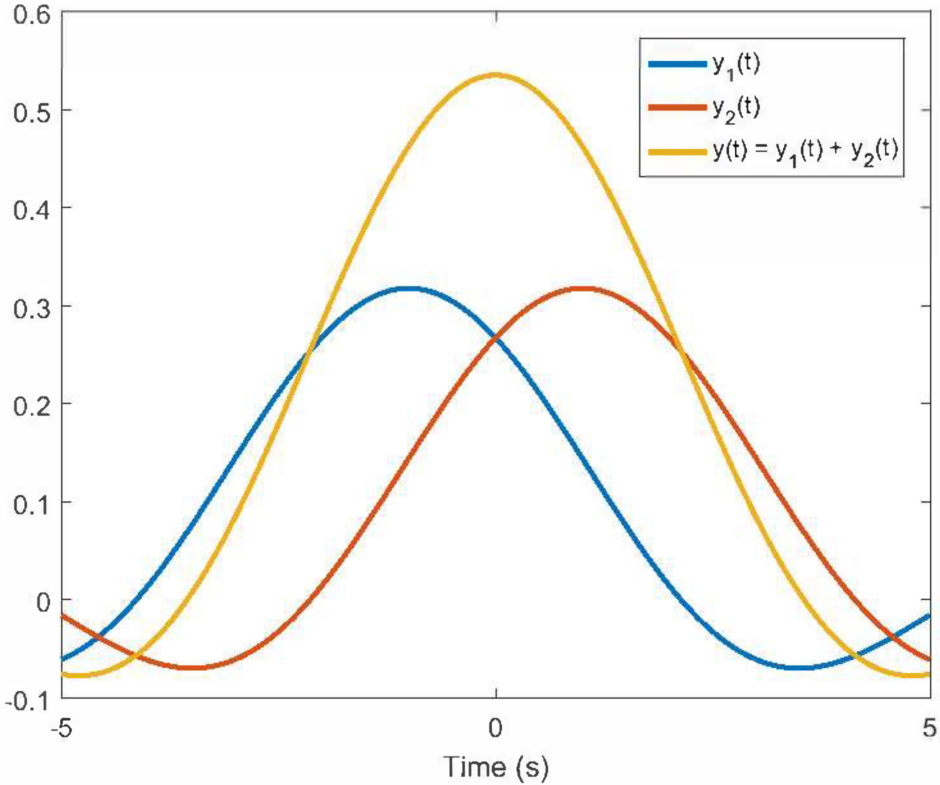
% Let's plot this, remembering that in MATLAB we need to divide the
% argument to sinc by pi to convert from the standard definition used in
% our textbook and elsewhere.

dt = 0.01;
t = -5 : dt : 5;
y1 = sinc((t+1)/pi)/pi; % break it into 2 parts
y2 = sinc((t-1)/pi)/pi;
y = y1+y2; % overall y(t)

figure
plot(t,y1,t,y2,t,y),legend('y_1(t)','y_2(t)','y(t) = y_1(t) + y_2(t)')
xlabel('Time (s)')

Eyt = sum(y.^2) * dt; % definition of energy, Riemann approximation
dW = 0.01; % frequency step for Parseval's energy integral
EyW = sum((2*cos(-1:dW:1)).^2) / (2*pi) * dW; % Definition of energy using
Parseval

fprintf('Energy of y(t) is %g using the time-domain Riemann approach.\n',
Eyt)
fprintf('Error of finding energy using Parseval (power spectrum) is %g%%.\n',
(EyW-Eyt)/Eyt*100)
```



0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-5
0
5
Time (s)
y1(t)
y2(t)
y(t) = y1(t) + y2(t)