EE-3032, HW-7

Effect of bandlimiting a signal

Problem from the Chaparro text:

- 8.4 (p. 529)
    - "Bandlimited," for a real signal, means that Y(Ω) is 0 for |Ω| > $\Omega_{H,}$ for some Ω_H. It also means that it is 0 for |Ω| < $\Omega_L \geq 0$, but that second part of the definition is not useful in this problem.
    - In (c), the Fourier Transform integral may take several steps, but none of them should get too complicated. Some hints.
        - If you get to mixed sinusoids and complex exponentials, it is probably easier to work with the sinusoids in this problem.
        - Recall that an even times and odd function is an odd function.
        - Recall that integrating an odd function over an interval symmetric about 0 yields 0.
        - Recall that sinc(x) = sin(x)/x is an even function.
- ***The following items are not required***, but try them if you want to go deeper into this problem:
    - It is not required, but it is interesting to plot y(t) and its constituent parts and compare them with the original x(t).
        - Note that the input pulses are 2 seconds apart (a frequency of 0.5 Hz) and the bandwidth of the system is 1 radian / s or 1/(2π) ≈ 0.16 Hz. Your graph and its parts make an important point about resolution that we'll explore further in EE3221.
            - For a counterexample from popular media, see https://www.youtube.com/watch?v=I_8ZH1Ggjk0
    - It is not required, but it is interesting to calculate the energy of y(t) using both the time-domain and the frequency domain.
        - You can calculate these integrals numerically in MATLAB using methods shown earlier in the class.
    - It is not required, but X(Ω) can be used to show that x(t) has infinite energy, but it curiously meets the dual criteria of a finite power signal.
        - If you think x(t) looks like it has finite energy, consider that the integral of $\delta^2(t)$ diverges. This can be shown by using either the triangular or rectangular pulse approximants to δ(t), squaring them, and then taking the same limit that transforms the original approximant function into δ(t).