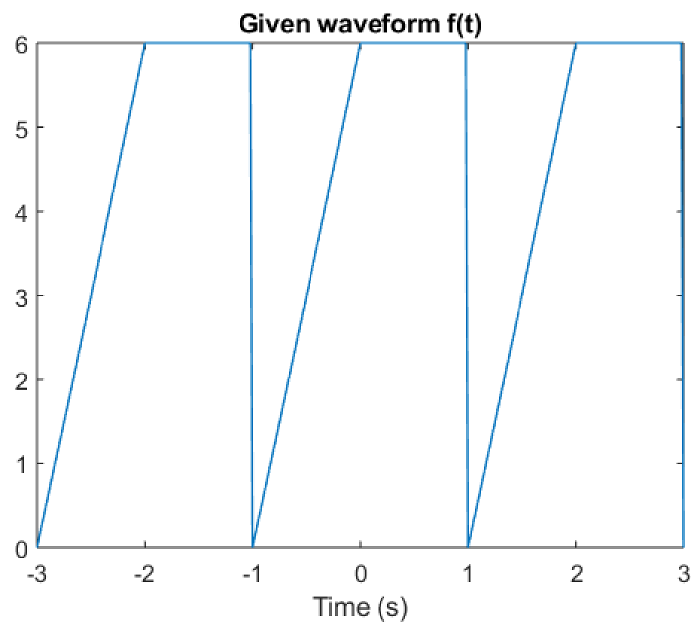
EE3032-2, Winter, 2019-'20, Dr. Durant's Homework 6, Problem 5.8 with power calculation additions

(a) Determine if the waveform has dc, even, or odd symmetry.

First, we'll plot the waveform.

```
A = 6;
T0 = 2; % seconds
dt = T0/100;
t = (-1.5*T0) : dt : (1.5*T0);
tBase = mod(t,T0); % calculate corresponding time on [0,T0) to simplify building the function
f = zeros(size(t));
f(tBase<=1) = A; % flat portion
f(tBase>=1) = A*tBase(tBase>=1) - A; % positive slope version mx+b format
figure,plot(t,f),title('Given waveform f(t)'),xlabel('Time (s)')
```



We can see from the graph that there is a non-0 DC term (since the average isn't 0), but that there is neither even nor odd symmetry (since flipping the graph from left to right about t=0 yields neither the function nor its negative),

(b) Obtain its cosine/sine Fourier series representation.

To find the a and b terms, we apply the integrals in Table 5-3 on page 206.

a0 is the average value of the signal. The area is (1/2)*1*A + 1*A = 3*A/2 using the area of a triangle and a rectange. The period is 2, so a0 = (3*A/2)/2 = 3*A/4.

The other coefficients can be simplified by breaking f into its 2 piecewise components on [-1,0] and [0,1].

$\omega_0 = \pi$

We show 2 methods. First, we have MATLAB do the symbolic integration, but we leave out the constant scaling factor A for simplicity...

```
f0 = 1/T0;
w0 = 2*pi*f0;
fprintf('The fundamental frequency w0 is %g radians/sec or %g Hz.\n',w0,f0)
```

```
The fundamental frequency w0 is 3.14159 radians/sec or 0.5 Hz.
```

```
tHold = t; % preserve samples of t so we can use symbolic t here
syms t
syms n integer
f1 = t+1; f2 = 1; % piecewise definition of f(t) on [-1,0] and [0,1]

disp('a_n, indefinite integrals of piecewise segments, followed by definite integrals and their sum, followed by values')
```

```
a_n, indefinite integrals of piecewise segments, followed by definite integrals and their sum, followed by values
```

```
as1 = int(f1*cos(n*pi*t),t)
```

as1 =

$$\frac{\cos(\pi\, n\, t) + \pi\, n\, (\sin(\pi\, n\, t) + t\sin(\pi\, n\, t))}{n^2\, \pi^2}$$

```
as2 = int(f2*cos(n*pi*t),t)
```

as2 =

$$\frac{\sin(\pi\, n\, t)}{n\, \pi}$$

```
a1  = int(f1*cos(n*pi*t),t,-1,0) % definite integral directly
```

a1 =

$$\frac{2\sin\left(\frac{\pi\, n}{2}\right)^2 - \pi\, n\sin(\pi\, n)}{n^2\, \pi^2}$$

```
a2  = int(f2*cos(n*pi*t),t,0,1)
```

a2 = 0

```
a   = a1 + a2
```

a =

$$\frac{2\sin\left(\frac{\pi\, n}{2}\right)^2 - \pi\, n\sin(\pi\, n)}{n^2\, \pi^2}$$

```
a   = simplify(a)
```

a =

$$\frac{(-1)^{n+1}\, ((-1)^n - 1)^2}{2\, n^2\, \pi^2}$$

```
subs(a,n,1:10) % evalute for various n
```

ans =

$$\left(\frac{2}{\pi^2} \quad 0 \quad \frac{2}{9\, \pi^2} \quad 0 \quad \frac{2}{25\, \pi^2} \quad 0 \quad \frac{2}{49\, \pi^2} \quad 0 \quad \frac{2}{81\, \pi^2} \quad 0\right)$$

```
disp('Same for b_n')
```

Same for b_n

```
bs1 = int(f1*sin(n*pi*t),t)
```

bs1 =

$$\frac{\sin(\pi\, n\, t) - \pi\, n\, (\cos(\pi\, n\, t) + t\cos(\pi\, n\, t))}{n^2\, \pi^2}$$

```
bs2 = int(f2*sin(n*pi*t),t)
```

bs2 =

$$-\frac{\cos(\pi\, n\, t)}{n\, \pi}$$

```
b1  = int(f1*sin(n*pi*t),t,-1,0)
```

b1 =

$$\frac{\sin(\pi\, n) - \pi\, n}{n^2\, \pi^2}$$

```
b2  = int(f2*sin(n*pi*t),t,0,1)
```

b2 =

$$\frac{2\sin\left(\frac{\pi\, n}{2}\right)^2}{n\, \pi}$$

```
b   = b1 + b2
```

b =

$$\frac{2\sin\left(\frac{\pi\, n}{2}\right)^2}{n\, \pi} + \frac{\sin(\pi\, n) - \pi\, n}{n^2\, \pi^2}$$

```
b   = simplify(b)
```

b =

$$\frac{(-1)^{n+1}}{n\,\pi}$$

```
subs(b,n,1:10)
```

ans =

$$\left(\frac{1}{\pi}\ \ -\frac{1}{2\,\pi}\ \ \frac{1}{3\,\pi}\ \ -\frac{1}{4\,\pi}\ \ \frac{1}{5\,\pi}\ \ -\frac{1}{6\,\pi}\ \ \frac{1}{7\,\pi}\ \ -\frac{1}{8\,\pi}\ \ \frac{1}{9\,\pi}\ \ -\frac{1}{10\,\pi}\right)$$

```
t = tHold;
```

We note that even with simplify and constraints on the symbolic variables (e.g., that n is an integer) that MATLAB can't always find the simplest form. Summarizing the results:

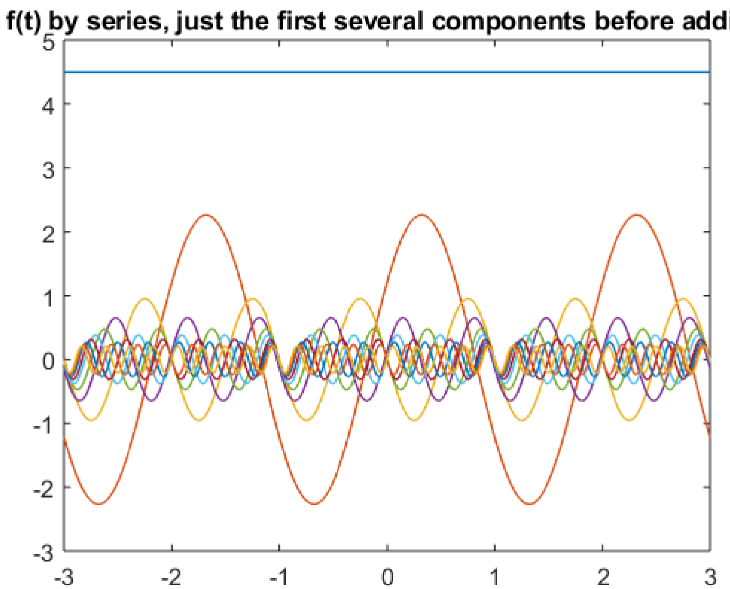
$a_n = \frac{2A}{(n\pi)^2}$, n>=1 and n odd; 0 for n even

$b_n = \frac{A \times -1^{n+1}}{n\pi}$, n>=1

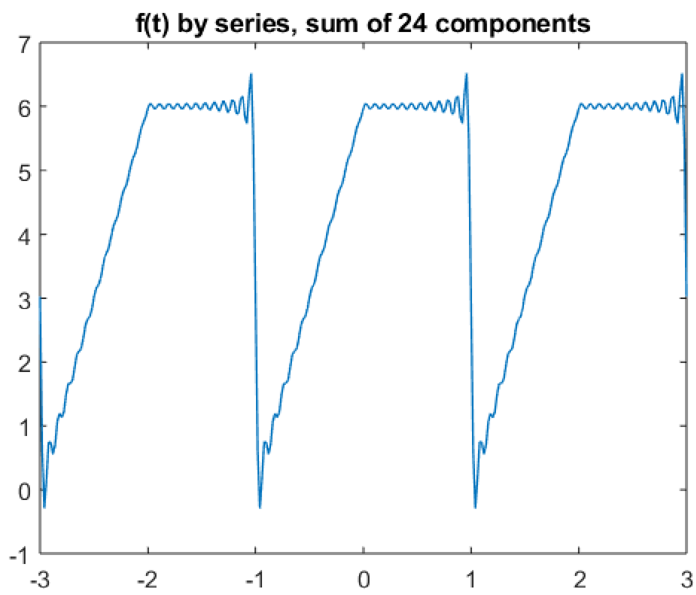
Let's skip ahead to part (e) to graphically confirm our answer is correct...

(e) Use MATLAB or MathScript to plot the waveform using a truncated Fourier series representation with n_{max}=100.

```
nmax = 23;
n = 0:nmax;
a = 2*A./((n*pi).^2); % for odd n >= 1
b = A./(n*pi).*(-1).^(n+1); % for odd n >= 1;
a(n==0) = 3*A/4; % a0 s an exception from the pattern
b(n==0) = 0; % b0 is unused (sin(0*t)), so set it to 0
a((mod(n,2)==0) & (n>0)) = 0; % even n besides 0
phase = (n'*w0) * t; % outer product, ' is transpose (row->column), rows of results are components; columns are time
components = (a' .* cos(phase)) + (b' .* sin(phase)); % column vector expands across columns of right argument when using .*
figure
plot(t,components(1:min(10,nmax+1),:)), title('f(t) by series, just the first several components before adding')
```




```
f_ser = sum(components); % sums the rows (components) into a single time series, doc sum to see how to sum other ways
figure
plot(t,f_ser), title(sprintf('f(t) by series, sum of %g components',length(n)))
```

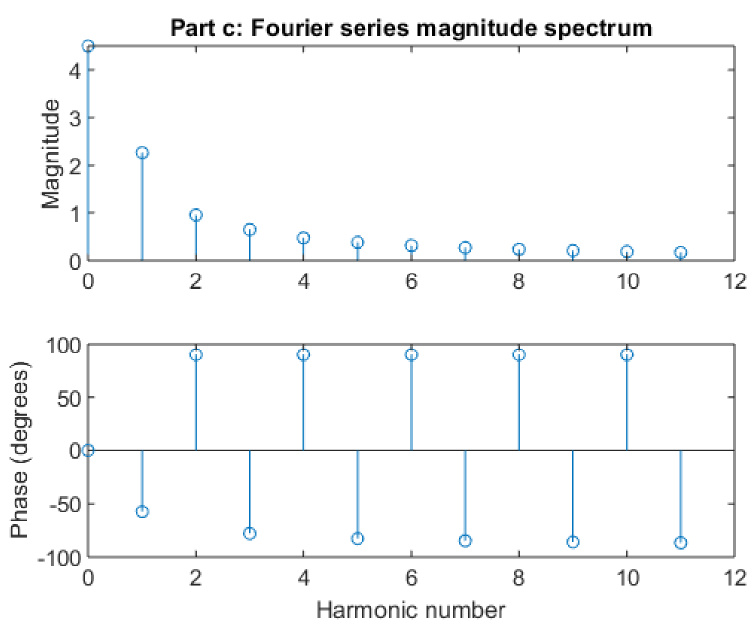


(c) Convert the representation to amplitude/phase format and plot the line spectra for the first five non-zero terms.

Use the equation for amplitude and phase from Table 5-3.

Note: this is sometimes called a "magnitude" spectrum, in contrast to an "amplitude" spectrum in which "amplitude" can carry a sign but phase is constrained to 2 quadrants, typically I and IV. Thus, "amplitude" matches the signs of the "a" coefficients.

```
c = hypot(a,b);
phi = -atan2(b,a);
idx = 12; % how many to plot; more than is asked for
figure
subplot(211),stem(n(1:idx), c(1:idx)),ylabel('Magnitude')
title('Part c: Fourier series magnitude spectrum')
subplot(212),stem(n(1:idx), rad2deg(phi(1:idx))),ylabel('Phase (degrees)')
xlabel('Harmonic number')
```



(d) Convert the representation to complex exponential format and plot the line spectra for the first five non-zero terms.

Here you would compute X = (a-jb)/2 for all n >= 1. X0 = a0. X_{-n} = X*_{n}.

Additional assigned problems: Also, calculate the power using at least 2 methods: ...

Method 1: directly from f(t)

```
fp = f(t>=0 & t<T0); % extract 1 period
```

```
Pd = (1/T0) * dt*sum(fp.^2); % Riemann sum for power integral
fprintf('Power using numeric integration is %g.\n', Pd)
```

```
Power using numeric integration is 23.8212.
```

Method 2: using Parseval's relation

```
in0 = n==0; % mask where n=0
Pp = a(in0).^2 + sum(a(~in0).^2 + b(~in0).^2)/2;
fprintf('Power using Parseval is %g.\n', Pp)
```

```
Power using Parseval is 23.9224.
```

Extra method: analytically

```
Pa = 4*(A^2)/3/T0; % Area under the f^2(t) in 1 period is 4(A^2)/3. Can be found by evaluating the integral analytically.
fprintf('Power analytically is %g.\n', Pa)
```

```
Power analytically is 24.
```