$\square$ green indicates required part of solution per problem statement.

(5.7) (p.385)

$\mathcal{F}\{\delta(t-\tau)\} = \int \delta(t-\tau) e^{-j\Omega t} dt = \boxed{e^{-j\Omega\tau}} = 1\angle(-\Omega\tau)$ ($\checkmark$ matches Table 5.2(2))

(a) $x(t) = \delta(t-1) + \delta(t+1)$

$X(\Omega) = e^{-j\Omega} + e^{j\Omega} = \boxed{2\cos(\Omega)}$

(b) $y(t) = \cos(\Omega_0 t) = \frac{1}{2}[e^{j\Omega_0 t} + e^{-j\Omega_0 t}]$

Duality of FT: $X(t) \leftrightarrow 2\pi x(-\Omega)$

$y(t) \leftrightarrow 2\pi Y(-\Omega)$

$Y(\Omega) = \cancel{2\pi y}$

$= 2\pi \cdot \frac{1}{2}[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$

$= \boxed{\pi(\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0))}$ ($\checkmark$ matches Table 5.2(10))

(c) $z(t) = \sin(\Omega_0 t) = \frac{-j}{2}[e^{j\Omega_0 t} - e^{-j\Omega_0 t}]$

$Z(\Omega) = 2\pi \cdot \frac{-j}{2}[\delta(-\Omega + \Omega_0) - \delta(-\Omega - \Omega_0)]$

$= \boxed{-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]}$ ($\checkmark$ matches Table 5.2(11))

(5.10) (a) $x(t) = \frac{1}{2}e^{-at}u(t) - \frac{1}{2}e^{at}u(-t)$, $a > 0$

$X(\Omega) = \underbrace{\frac{1}{2}\left(\frac{1}{j\Omega + a}\right)}_{\text{Table 5.2(7)}} - \underbrace{\frac{1}{2}\left(\frac{1}{-j\Omega + a}\right)}_{x(-t) \leftrightarrow X(-\Omega)}$

$= \frac{1}{2}\left(\cancel{(j\Omega - a) - (}\frac{(a - j\Omega) - (a + j\Omega)}{a^2 + \Omega^2}\right) = \boxed{\frac{-j\Omega}{a^2 + \Omega^2}}$

$Y(\Omega) = \lim_{a \to 0} X(\Omega) = \frac{-j\Omega}{\Omega^2} = \boxed{-j/\Omega}$

$u(t) = \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t) = \frac{1}{2} + y(t)$ (alternate way to build unit step)

$U(\Omega) = 2\pi\frac{1}{2}\delta(\Omega) - \frac{j}{\Omega} = \boxed{\pi\delta(\Omega) + \frac{1}{j\Omega}}$ ($\checkmark$ matches table 5.2(3))

5.11 (a) (i) $c(t) = \cos(10t)$ (carrier) $\quad C(\Omega) = \pi[\delta(\Omega-10)+\delta(\Omega+10)]$

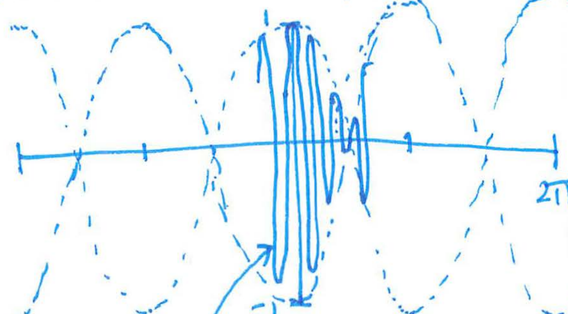
$m(t) = \cos(t)$ (message) $\quad M(\Omega) = \pi[\delta(\Omega-1)+\delta(\Omega+1)]$

$y(t) = m(t)c(t) = \cos(t)\cos(10t) = \frac{1}{2}[\cos(9t)+\cos(11t)]$ (trig. identity)

Sketch is a bit complex. Best approach is cos(t) (low freq.) envelope which is then mult. by cos(10t). Both functions are on $[-1,1]$, so cos(t) acts as a band or envelope.

← cos(t) envelope $(\max(m(t)) = 1)$

⟹ Sketch or software printout



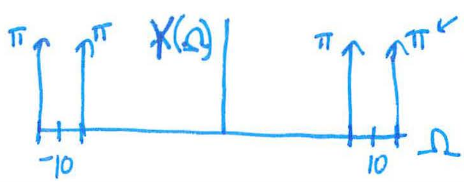
$-\cos(t)$, other half of envelope $(\min(m(t)) = -1)$

y(t), partially sketched

Table 5.2(14) gives $\underbrace{x(t)}_{m(t)}\underbrace{\cos(\Omega_0 t)}_{c(t)} \leftrightarrow \frac{1}{2}[X(\Omega-\Omega_0)+X(\Omega+\Omega_0)]$

← mathematically, m & c can be reversed, but it is easiest this way.

⟹ Sketch spectrum



← indicates area of impulse if not 1

(b) $E_x = \int |x(t)|^2 dt = \frac{1}{2\pi}\int |X(\Omega)|^2 d\Omega = \dots$

$x(t) = \frac{\sin(0.5t)}{\pi t} = \frac{1}{2\pi}\frac{\sin(0.5t)}{0.5t} = \frac{1}{2\pi}\operatorname{sinc}(0.5t)$

From Table 5.2(13), $X(\Omega) = u(\Omega+\frac{1}{2}) - u(\Omega-\frac{1}{2})$

$E_x = \dots = \frac{1}{2\pi}\int \left(u(\Omega+\tfrac{1}{2}) - u(\Omega-\tfrac{1}{2})\right)^2 d\Omega = \frac{1}{2\pi}\int u(\Omega+\tfrac{1}{2}) - u(\Omega-\tfrac{1}{2})\, d\Omega$

$= \frac{1}{2\pi}\cdot 1 = \boxed{\frac{1}{2\pi}}$