EE3032, Winter 2019-'20, Homework 5 Solutions, Dr. Durant

2.32 An LTI system has the impulse response $h(t) = 5e^{-t}u(t) - 16e^{-2t}u(t) + 13e^{-3t}u(t)$. The input is $x(t) = 7\cos(2t + 25°)$. Compute the output y(t).

x(t) is a single steady-state sinusoid, so it would be useful to have the system transfer function. Recognizing that h(t) is a linear combination (sum) of 3 causal decaying impulses, we can apply linearity and write the transfer function in 3 corresponding components: $H(\omega) = \frac{5}{1+j\omega} - \frac{16}{2+j\omega} + \frac{13}{3+j\omega}$. Let's have MATLAB calculate the H at the input frequency of 2 radians/s:

```
omega = 2;
Hparts = [5/(1+1j*omega), -16/(2+1j*omega), 13/(3+1j*omega)]
```

```
Hparts = 1×3 complex
   1.0000 - 2.0000i  -4.0000 + 4.0000i   3.0000 - 2.0000i
```

```
H = sum(Hparts)
```

```
H = 0
```

So, the transfer function is equal to 0 ("has a null") at the frequency of interest, yielding y(t) = 0.

2.37 We observe the following input-output pair for an LTI system:

- $x(t) = u(t) + 2\cos(2t)$
- $y(t) = u(t) - e^{-2t}u(t) + \sqrt{2}\cos(2t - 45°)$
- $x(t) \to \text{LTI} \to y(t)$

Determine y(t) in response to a new input $x(t) = 5u(t-3) + 3\sqrt{2}\cos(2t - 60°)$.

Using linear systems theory for the cosine component we calculate the gain |H| as Vout/Vin = sqrt(2)/2 or 1/sqrt(2) and see that the phase shift is -45 degrees. So, we have $H(2) = \frac{1}{\sqrt{2}}\angle{-45°}$

The leftover components of x(t) and y(t) tell us that the step response is an exponential decay from 0 to 1 with a time constant of 1/2 second as given by the first 2 terms of y(t).

We then recognize that the new input x(t) has 2 corresponding components. To the first we apply scaling and time shift to the step response and to the second we apply the gain and phase shift form the H above, yielding $y(t) = 5(u(t-3) - e^{-2(t-3)}u(t-3)) + 3\cos(2t - 60° - 45°)$

$= 5(1 - e^{-2t+6})u(t-3) + 3\cos(2t - 105°)$

5.1(b) A system is characterized by the differential equation $c_1 \frac{dy}{dt} + c_2 y = 10\cos(400t - 30°)$. Determine y(t), given that $c_1 = 10^{-2}$ and $c_2 = 0.3$.

Call the righthand side of the equation x(t). By temporarily letting $x(t) = e^{-j\omega t}$, we can solve for the system transfer function implicitly as we did in class: $c_1 j\omega H(\omega) + c_2 H(\omega) = 1$ where the common multiplying term x(t) has been omitted from each of the 3 terms. For the complex exponential, multiplication by $j\omega$ happens when a derivative is taken and multiplication by the complex gain $H(\omega)$ occurs between the input and the otuput. Solving the equation, we have $H(\omega) = \frac{1}{c_1 j\omega + c_2}$. Substituting in the 3 parameters, we have

```
omega = 400;
c1 = 1e-2; c2 = 0.3;
H = 1/(c1*1j*omega+c2)
```

```
H = 0.0186 - 0.2486i
```

```
H_mag = abs(H)
```

```
H_mag = 0.2493
```

```
H_phase_deg = rad2deg(angle(H))
```

```
H_phase_deg = -85.7108
```

We then apply the transfer function's gain and phase shift to the input, yielding...

```
y_mag = 10 * H_mag
```

```
y_mag = 2.4930
```

```
y_phase_deg = -30 + H_phase_deg
```

```
y_phase_deg = -115.7108
```

$y(t) = 2.4930\cos(400t - 115.7108°)$