EE3032-4 HWK4 Fall, 2019 Dr. Durant

2.28 $h(t) = x(t) = 2\cos(t)\,u(t)$. Find $y(t)$ & determine stability.

$$y(t) = h(t) * x(t) = \int h(t-\tau)\,x(\tau)\,d\tau = \int 2\cos(t-\tau)\,u(t-\tau)\,2\cos(\tau)\,u(\tau)\,d\tau$$

$$= 4\int_0^t \cos(t-\tau)\cos(\tau)\,d\tau = 4\int_0^t \frac{1}{2}\left(\cos(t-\tau+\tau) + \cos(t-\tau-\tau)\right)d\tau$$

$$\cancel{= 2\cos(t)\int_0^t \cos(t-2\tau)\,d\tau = -\cos(t)\sin(t-2\tau)\Big|_{\tau=0}^{t} = -\cos(t)\left(\sin(-t) - \sin(t)\right)}$$

$$\cancel{= t + 2\cos(t)\sin(t)}$$

$$= 2\int_0^t \cos(t) + \cos(t-2\tau)\,d\tau$$

$$= 2\left[\tau\cos(t) - \frac{1}{2}\sin(t-2\tau)\right]\Big|_{\tau=0}^{t}$$

$$= 2\left[t\cos(t) - \frac{1}{2}\sin(-t) + \frac{1}{2}\sin(t)\right]$$

$$= \underline{2t\cos(t)} + 2\sin(t)$$

↑ grows w/o bound ∴ unstable

2.29(a,c,d) $H(w) = \frac{1}{jw+3}$

(a) $x(t) = 3 \therefore w = 0 \therefore H(0) = \frac{1}{3} \therefore y(t) = 1$

(c) $x(t) = 5\cos(4t)$, $w = 4$, $H(4) = \frac{1}{3+4j} = 0.12 - j0.16 = 0.2\angle -0.9273$

$\uparrow$
-53.13°

y: amplitude $= 5 \cdot 0.2 = 1$, $\phi = \emptyset + \angle H = -53.13^\circ$

$y(t) = \cos(4t - 53.13^\circ)$

$\uparrow$
w unchanged

(d) $x(t) = \delta(t)$

$H(w) = \frac{1}{3+jw} = \frac{1}{a+jw} \Leftrightarrow h(t) = e^{-3t}u(t)$

$y(t) = x(t) * h(t) = \delta(t) * (e^{-3t}u(t)) = e^{-3t}u(t)$

2.30 (a, b) $\ddot{y} + 2\dot{y} + 7y = 5\dot{x}$ $\quad x(t) = \cos(\omega t)$

(a) $\omega = 2$, find $y(t)$. In phasor domain $\frac{d}{dt} \to (j\omega)$

$$(j\omega)^2 Y + 2j\omega Y + 7Y = 5j\omega X$$

$$Y(7 - \omega^2 + 2j\omega) = 5j\omega X$$

$$Y = \frac{5j\omega}{7-\omega^2+2j\omega} X = HX$$

$$H = \frac{5j2}{7-2^2+2j2} = \frac{j10}{3+j4} = 1.6 + j1.2 = 2\angle \underbrace{0.6435}_{36.8699^\circ}$$

$$y(t) = \underbrace{2}_{|H|}\cos(\underbrace{2}_{\omega}t + \underbrace{36.8699^\circ}_{\angle H})$$

(b) $H = \frac{5j\omega}{7-\omega^2+2j\omega} \cdot \frac{j}{j} = \frac{-5\omega}{(7-\omega^2)j - 2\omega} = \frac{5\omega}{2\omega - j(7-\omega^2)}$

$\phi = 0 \to$ denominator real $\to 7 - \omega^2 = 0 \to \boxed{\omega = \pm\sqrt{7}}$

(cosine is <u>even</u>, so $\omega = -\sqrt{7}$ is <u>same</u> as $\omega = +\sqrt{7}$.)

2.36

ω: 0 1 2

$x(t) = 1 + 2\cos(t) + 3\cos(2t)$

$y(t) = 6\cos(t) + 6\cos(2t)$

$H = \frac{y}{x}$ 0 $\frac{6\angle 0}{2\angle 0} = 3$ $\frac{6\angle 0}{3\angle 0} = 2$

Multiply amplitudes

$x(t) = 4 + 4\cos(t) + 2\cos(2t)$

$y(t) = 12\cos(t) + 4\cos(2t)$

5.2(a) $c_1\ddot{y} + c_2\dot{y} + c_3 y = x = A\cos(\omega t + \phi)$

$$c_1(j\omega)^2 Y + c_2 j\omega Y + c_3 Y = X$$

$$Y\left((c_3 - c_1\omega^2) + j c_2 \omega\right) = X$$

$$Y = \frac{1}{c_3 - c_1\omega^2 + j c_2 \omega} X = HX$$

$$= \frac{1}{3 - 10^{-6}(10^3)^2 + j\,3\times10^{-3}(10^3)} X$$

Corrected j sign 10/24/2019 @ 1100

$$= \frac{1}{3 - 1 + j3} X = \frac{1}{2 + j3} X$$

$$H = \frac{1}{2+j3} = 0.1538 - j0.2308 = \underset{\substack{\uparrow \\ 1/\sqrt{13}}}{\underline{0.2774}} \angle \underset{\substack{\uparrow \\ -56.3099^\circ}}{-0.9828}$$

$$y(t) = (A \cdot |H|)\cos(\omega t + \phi + \angle H) = (12 \cdot 0.2774)\cos(10^3 t + 60^\circ - 56.3099^\circ)$$

$$= 3.3282\cos(1000t + \underset{\substack{\cancel{1.03}\ \text{radians} \\ 0.0644}}{\underline{\overset{3.6901}{\cancel{116.3099}}^\circ}})$$