(4.1) (p.318)

(a) $y(t) = x^2(t)$ $x(t) = e^{j\pi t/4}$, which is an eigenfunction of an LTI system

$y(t) = x^2(t) = e^{j\pi t/2}$ Ω changed ∴ eigenfunction property did not hold, as expected for non-linear system

(b) $y(t) = x(t)(u(t) - u(t-1))$ $x(t) = e^{j\pi t/4}$

$y(t) = e^{j\pi t/4}(u(t) - u(t-1))$. This is not an (eternal) complex exponential ∴ eigenfunction property did not hold, as expected for a time-varying function

(4.2) $y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\,d\tau$ $x(t) = e^{j\Omega_0 t}$. We expect: $y(t) = e^{j\Omega_0 t} H(j\Omega_0)$.

(a) $y(t) = \frac{1}{T}\int_{t-T}^{t} e^{j\Omega_0 \tau}\,d\tau = \frac{1}{T}\frac{1}{j\Omega_0} e^{j\Omega_0 \tau}\Big|_{\tau=t-T}^{t} = \cancel{\frac{-j}{T\Omega_0} e^{j\Omega_0(t-(t-T))}} = \cancel{-je^{j\Omega_0}}$

$= \frac{-j}{T\Omega_0}\left(e^{j\Omega_0 t} - e^{j\Omega_0 t}e^{-j\Omega_0 T}\right) = \underbrace{\frac{-j(1-e^{-j\Omega_0 T})}{T\Omega_0}}_{H(j\Omega_0)} e^{j\Omega_0 t}$

IF input is specifically $j\Omega_0$

$H(j\Omega_0) = \frac{1-e^{-j\Omega_0 T}}{Tj\Omega_0}$ } book's answer

$H(x) = \frac{1-e^{-xT}}{Tx}$

(b) To find $h(t)$, let $x(t) = \delta(t)$. Then $y(t) = h(t)$.

$h(t) = \frac{1}{T}\int_{t-T}^{t} \delta(\tau)\,d\tau = \frac{1}{T}(u(t) - u(t-T))$

(nothing else to do since HWK said to skip Laplace part.)

(4.3a) $x_1(t) = 1 + \cos(2\pi t) - \cos(6\pi t)$

$x_2(t) = 1 + \cos(2\pi t) - \cos(6t)$

(i) fundamental period

x_1 : $\Omega_1 = 2\pi$ $\quad \Omega_2 = 6\pi$

$f_1 = 1$ $\quad F_2 = 3$

$T_1 = 1$ $\quad T_2 = 1/3$

$T_0 = 1$

$\boxed{\Omega_0 = 2\pi}$

periodic

x_2 : $\Omega_1 = 2\pi$ $\quad \Omega_2 = 6$

$F_1 = 1$ $\quad F_2 = \frac{3}{\pi}$

$T_1 = 1$ $\quad T_2 = \pi/3$

$LCM(T_1, T_2) \to \infty$ due to irrational ratio

$\therefore \Omega_0 \to 0$, does not exist

not periodic

(ii) $x_1(t) = 1 + \cos(\Omega_0 t) - \cos(3\Omega_0 t)$

$c_0 = 1$ $\quad c_1 = 1/2$ $\quad k=1$ $\quad -1/2$ $\quad k=3$

The division by 2 for $k>0$ follows from the form in class & book:

$$\ldots + 2\sum_{k=1}^{\infty} c_k \cos(\Omega_0 k t) + \ldots$$

Extra: $P_{x_1} = \frac{1}{T_0}\int_{T_0} x_1^2(t)\,dt = 1 + \frac{1}{2} + \frac{1}{2} = 2\ W$

DC $\quad$ cos of any freq. as derived in class

Compare w/. Parseval. $k>0 \to$ Need to scale c_k, d_k by $\sqrt{2}$ to get pwr correct

$$P_{x_1} = c_0^2 + \sum_{k=1}^{\infty} (\sqrt{2}\,c_k)^2 = 1^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1 + \frac{1}{2} + \frac{1}{2} = 2W \checkmark$$

(iii) for x_2, Ω_0 does not exist. There is no way to find $k \in \mathbb{Z}^+$ for each sinusoidal component.