EE3032 HW-3 F'17 Page 1/4 Dr. Dorant

- 2.14 (p.171)
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- Additional problem

2.14 $h(t) = u(t) - u(t-1)$

a) Input $= x(t)$. Is output $y(t) = \int_{t-1}^{t} x(\tau)\,d\tau$?

Find output w/ convolution. Use h-folded version since h is simpler.

$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\,d\tau \rightarrow h = 0$ almost everywhere, except

$0 < t - \tau < 1$ 0,1 are limits from $h(\cdot)$

$-t < -\tau < 1 - t$

$t > \tau > t-1$

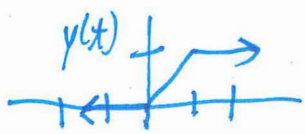
$= \int_{t-1}^{t} x(\tau)\,d\tau$

Q.E.D.

b) $x(t) = u(t)$

$$y(t) = \begin{cases} 0, & t<0 \text{ (no overlap)} \\ t, & 0<t<1 \text{ (partial overlap of } h \,\&\, t\text{, so evaluate: } \int_{0}^{t} 1\,d\tau = t) \\ 1, & t>1 \quad (= \int_{t-1}^{t} 1\,d\tau = 1) \end{cases}$$

$t-1<0 \Rightarrow x(\tau) = 0$, so omit



c) Step response is output when input is unit step, so we solved in b).

$$s(t) = \begin{cases} 0, & t<0 \\ t, & 0<t<1 \\ 1, & t>1 \end{cases} = r(t) - r(t-1) = t\,u(t) - (t-1)u(t-1)$$

System is LTI (since $h(t)$ exists) $\therefore$ linear, so $\int \,\&\, \frac{d}{d\tau}$ commute w/ system.

Let input be derivative of original input:

$y(t) = \int \left(\frac{d}{dt} x(\tau)\right) h(t-\tau)\,d\tau = \frac{d}{dt}\left(\int x(\tau) h(t-\tau)\,d\tau\right) = \frac{d}{dt}(s(t))$

Let $x(t) = u(t)$, then $\frac{d}{dt}x(t) = \delta(t)$, then $y(t) = \underline{h(t) = \frac{d}{dt}s(t)}$

$\therefore$ The impulse response, $h(t)$, is the derivative of the step response $s(t)$.

$h(\cdot) = 1$ if $0 < t - \gamma < 1$

$-t < -\gamma < 1 - t$

$t > \gamma > t - 1$ → limits of integration

2.16 $h(t) = u(t) - u(t-1)$ $\quad x(t) = \sum_{k=0}^{9} \delta(t - kT)$

(a) $y(t) = \int x(\gamma) h(t-\gamma)\, d\gamma = \int \sum_k \delta(\gamma - kT)\big(u(t-\gamma) - u(t-\gamma-1)\big)\, d\gamma$

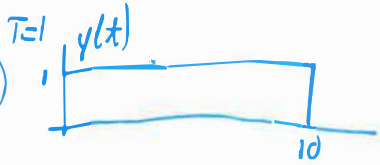
$= \sum_k \int_{t-1}^{t} \delta(\gamma - kT) \cdot \underset{\uparrow}{1}\, d\gamma = \boxed{\sum_k \big(u(\underset{t-1-kT}{\uparrow}) - u(\underset{t-kT}{\uparrow})\big)} \overset{\text{if } T=1}{\stackrel{?}{=}} \boxed{u(0) - u(10)}$

value of $h(\cdot)$ given γ

general solution

add up 10, 1x1 pulses starting @ 0, 1, …, 9.

(b) $T=1$



(c) $T = 0.5$. The pulses now overlap.

One approach: all even k: 0, 2, 4, 6, 8 → $y_1(t) = u(t-0) - u(t-5)$

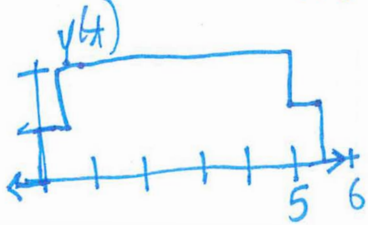
start @ $kT = 0$ $\quad$ start @ $kT = 4$

all odd k: 1, 3, 5, 7, 9 → $y_2(t) = u(t-0.5) - u(t-5.5)$

start @ $kT = \frac{1}{2}$ $\quad$ start @ $kT = 4\frac{1}{2}$

$y(t) = y_1(t) + y_2(t) = \cancel{u(t) + u(0.5) - u(t}$

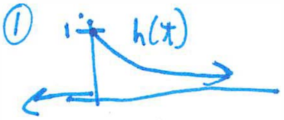
$= u(t) + u(t-0.5) - u(t-5) - u(t-5.5)$

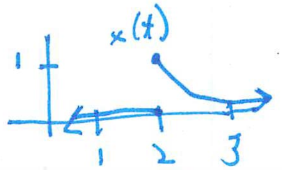


Additional Problem

$h(t) = e^{-t} u(t)$ $\quad x(t) = e^{-(2(t-2))} u(t-2)$

①





② $y(t) = \int x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} e^{-2(\tau-2)} u(\tau-2)\, e^{-(t-\tau)} u(t-\tau)\, d\tau$

Graphically, from ①, we see after folding $h(\cdot)$ that there is no overlap until it is shifted right by $t=2$. The overlap, then, for all $t \geq 2$, occurs from $\tau = 2$ to t.

$$y(t) = \begin{cases} 0, & t < 2 \\ \int_2^t e^{-2(\tau-2)} e^{-(t-\tau)}\, d\tau = y_1(t), & t \geq 2 \end{cases}$$

$$y_1(t) = \int_2^t e^{-2\tau+4-t+\tau}\, d\tau = \int_2^t e^{4-t-\tau}\, d\tau = -\left[e^{4-t-\tau}\right]_{\tau=2}^{\tau=t}$$

$$\cancel{= -\left[0 - e^{4-t-2}\right] = e^{4-t-2} = e^{2-t}}$$

~~use limit~~

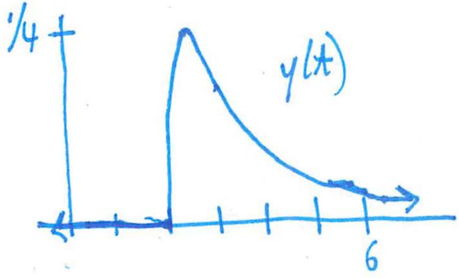
$$= -\left(e^{4-2t} - e^{4-t-2}\right) = e^{2-t} - e^{2(2-t)}$$

Check: If $t = 2^+$, $y_1(t) = e^{2-2} - e^{2(2-2)} = 1 - 1 = 0$
If $t = 2^-$, $y(t) = 0$

$\therefore$ $y(t)$ is 0th orde continuous, as it must be since no δs are involv

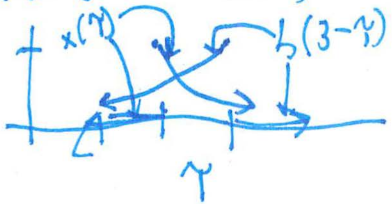
So, $$y(t) = \begin{cases} 0, & t \leq 2 \\ e^{2-t} - e^{2(2-t)}, & t \geq 2 \end{cases}$$

(3)



(4) vs. τ, $t=3$

$x(\tau) = e^{-(2(\tau-2))} u(\tau-2)$ $\quad$ $h(t-\tau) = e^{-(t-\tau)} u(t-\tau) = e^{-(3-\tau)} u(3-\tau)$



The product of $x(\tau)$ & $h(3-\tau)$ is 0 unless $2<\tau<3$.

$y(3)$ = area under product from $-\infty$ to ∞,

which = area under product from 2 to 3.