EE3032-4 - Dr. Durant - HWK 2 - 3 Oct. 2019

~~2.10(b)~~

1.33 (a,b)

ⓐ $x_1(t) = 3[u(t+2) - u(t-2)]$

$\boxed{\text{Energy}}$ since $\int_{-\infty}^{\infty} x_1^2(t)\,dt = \int_{-2}^{2} 3^2\,dt = 36 \underbrace{< \infty}_{}$

informal notation for "is finite"

ⓑ $x_2(t) = 2[r(t) - r(t-2)]$

$\boxed{\text{Power}}$ $E = \int_{-\infty}^{\infty} x_2^2(t)\,dt > \int_{2}^{\infty} 4\,dt \to \infty$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x_2^2(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \underbrace{16u(t)\,dt}_{\text{Simplify, modify on } [0,2], \text{ if finite, doesn't affect limit} \to \infty} = 16 \lim_{T\to\infty} \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} u(t)\,dt}_{\text{average value of } u(t) \text{ over } [-\alpha,\alpha] = \frac{1}{2}} = 16\cdot\frac{1}{2} = 8 < \infty$$

1.35 (a) $x_1(t) = [1-\underbrace{e^{2t}}_{}]u(t)$.

Diverges to ∞, so $E \to \infty$, Check P.

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} \underbrace{1 - 2e^{2t} + e^{4t}}_{\text{square}}\,dt$$

↑ due to $u(t)$

. Dominated by $\frac{1}{T}\int_0^{T/2} e^{4t}\,dt = \frac{1}{4T}e^{4t}\Big|_0^{T/2} = \frac{1}{4T}(e^{2T}-1)$

$\lim_{T\to\infty}$ diverges → $\boxed{\text{neither}}$

↑ not power, either

1.38(b) $x_2(t) = e^{-a|t|}$, $\underbrace{a > 0}_{\text{ensures exp. decay on both sides}}$. $E_x = \int_{-\infty}^{\infty} \underbrace{e^{-2a|t|}}_{\text{square of mag since } x_2 \text{ is non-negative}}\,dt = \underbrace{2\int_0^{\infty}}_{\text{symmetry (even)}} e^{-2at}\,dt = -\frac{1}{a}e^{-2at}\Big|_0^{\infty} = \frac{-1}{a}(0-1) = \boxed{1/a}$

↑ $t = |t|$ if $t \geq 0$

2.1(b,e)

ⓑ $y(t) = \underbrace{3\sin(t)}_{\text{time-varying gain}}\,x(t)$. $\boxed{\text{Linear, but not TI.}}$

↑ Linear: Let $x(t) = c_1x_1(t) + c_2x_2(t)$

$$y(t) = 3\sin(t)[c_1x_1(t) + c_2x_2(t)]$$
$$= 3\sin(t)c_1x_1(t) + 3\sin(t)c_2x_2(t)$$
$$= \underbrace{c_1}_{\text{checks scaling property}}3\sin(t)x_1(t) + c_2\underbrace{3\sin(t)x_2(t)}_{\text{sys applied to } x_2 \to y_2(t)} = c_1\overbrace{y_1(t)}^{\text{Means system applied to } x_1(t)} + c_2y_2(t)$$

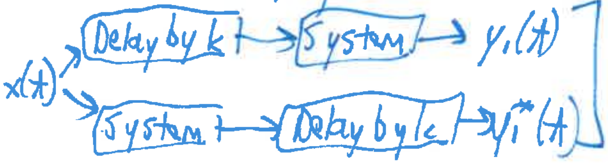
TI? let $x_1(t) = x(t-k)$

$y_1(t) = 3\sin(t)x(t-k)$ ← delayed x_1 into system

$y_1^*(t) = y(t-k) = 3\sin(t-k)x(t-k)$ ← delay output $y(t)$ applied first to $x(t)$

$y_1(t) \overset{?}{=} y_1^*(t)$

No! since $\sin(t) \neq \sin(t-k)$ for any k.



TI means these 2 are equal

2.1(e) $y(t)=\int_{-\infty}^{t} x(\tau)d\tilde{\tau}$. $\boxed{\text{Linear \& TI.}}$

↑
Can Prove as in b,
$cx(t)$ & $x_1(t)+x_2(t)$
Interchange w/ integration
∴ integration is linear

TI: Delay input:

$\overset{*}{y}(t)=\int_{-\infty}^{t} x(\tau-k)d\tau$

$\sigma=\tau-k$ $d\sigma=d\tau$

$$\overset{*}{y}(t)=\int_{-\infty}^{t-k} x(\sigma)d\sigma=y(t-k)$$

↑↗ dummy var. of integration

↖ Sub $t\to t-k$ in definition

2.2(a,e)

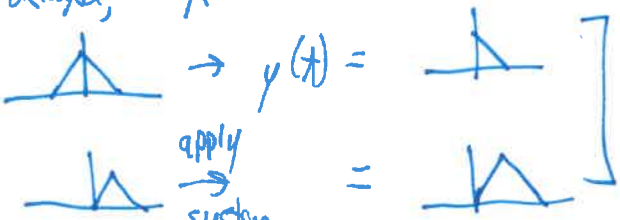
ⓐ $y(t)=3x(t-1)$ is LTI. Can prove as above

ⓔ $y(t)=x(t)u(t)$ is L but not TI.

TI? If input is delayed, shape of output changes.

E.g. let $x(t)=$

Delay $x(t)$



These are not time-shifted replicas. They are truncated differently.

2.5(a,c) $s(t)=r(t)-2r(t-1)+r(t-2)$

↑
step response

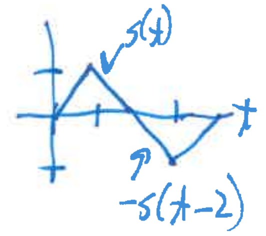
Shortcut. Express $x(t)$ in terms of $u(t)$.
System changes each u to s, just as it changes δ to h!
That is definition of step/impulse response

ⓐ $x_1(t)=u(t)-u(t-2) \therefore y_1(t)=s(t)-s(t-2)=(r(t)-2r(t-1)+\cancel{r(t-2)})$

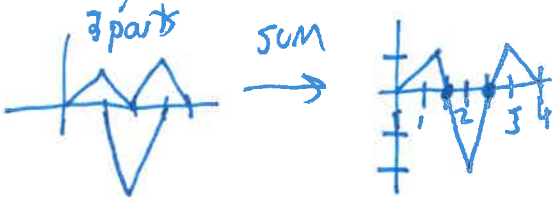
$\pm(\cancel{r(t-2)}\pm 2r(t-3)\mp r(t-4))$

swap signs

$y_1(t)$



ⓒ $x_3(t)=u(t)-2u(t-1)+u(t-2)\rightarrow y_3(t)=s(t)-2s(t-1)+s(t-2)$



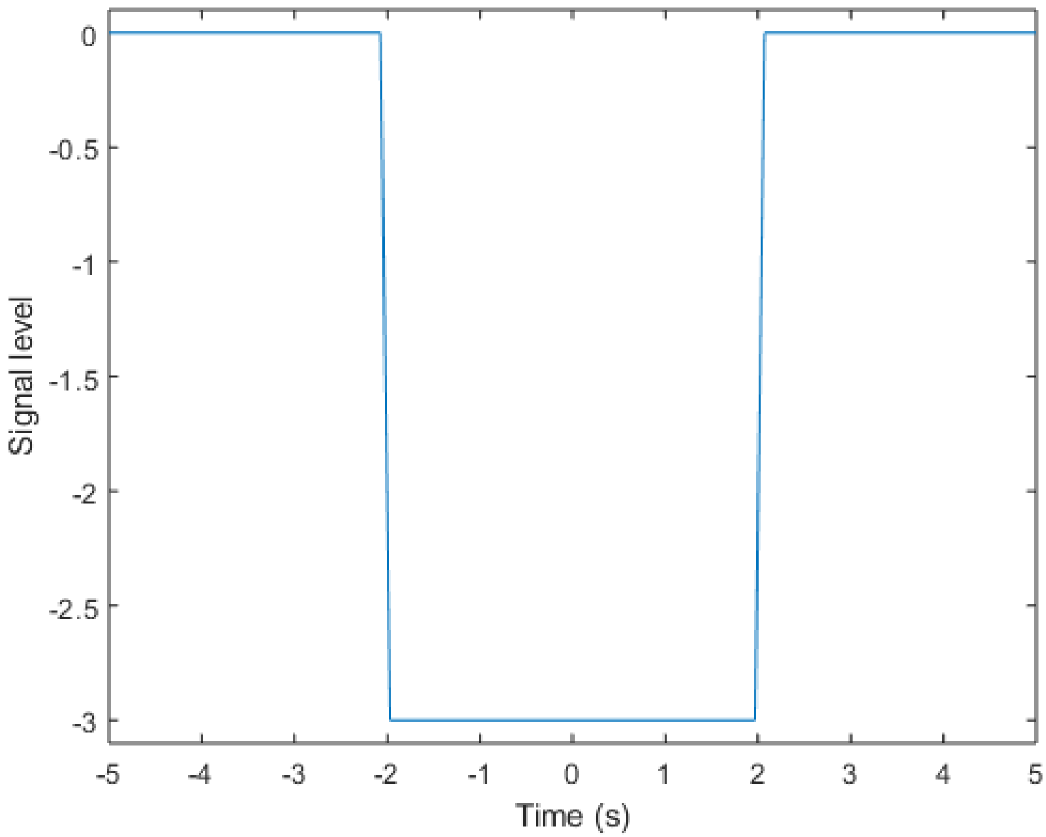
EE3032: Dr. Durant's solution to Problem 1.33, 10/7/2019

Determine if each of the following signals is a power signal, an energy signal, or neither.

b. $x_2(t) = 3[u(t-2) - u(t+2)]$

This is the negative of part (a), so it is still an energy signal.

```
t = linspace(-5, 5);
x2 = 3 * ((t>2) - (t>-2));
plot(t, x2)
xlabel('Time (s)'), ylabel('Signal level')
ylim([min(x2)-0.1, max(x2)+0.1]) % so flat areas don't hit plot border
```



We can see that all of the non-zero values of the signal are captured in the variable, so we can calculate the energy as the integral of the magnitude squared of the signal. For a real signal, the square is the same as the magnitude squared. We expect this to be 4 s × $3^2 = 36$.

```
dt = diff(t(1:2));
E = sum(dt * x2.^2);
fprintf('The energy is %g.\n', E)
```

```
The energy is 36.3636.
```

This is off by about 1% from the true value, which is expected since we're making an approximation with a finite number of time values.

```
N = length(t);
fprintf('%g values have been calculated in the function.\n', N)
```

```
100 values have been calculated in the function.
```

You can pass a 3rd argument to specify how many values to calculate instead of the default of 100.

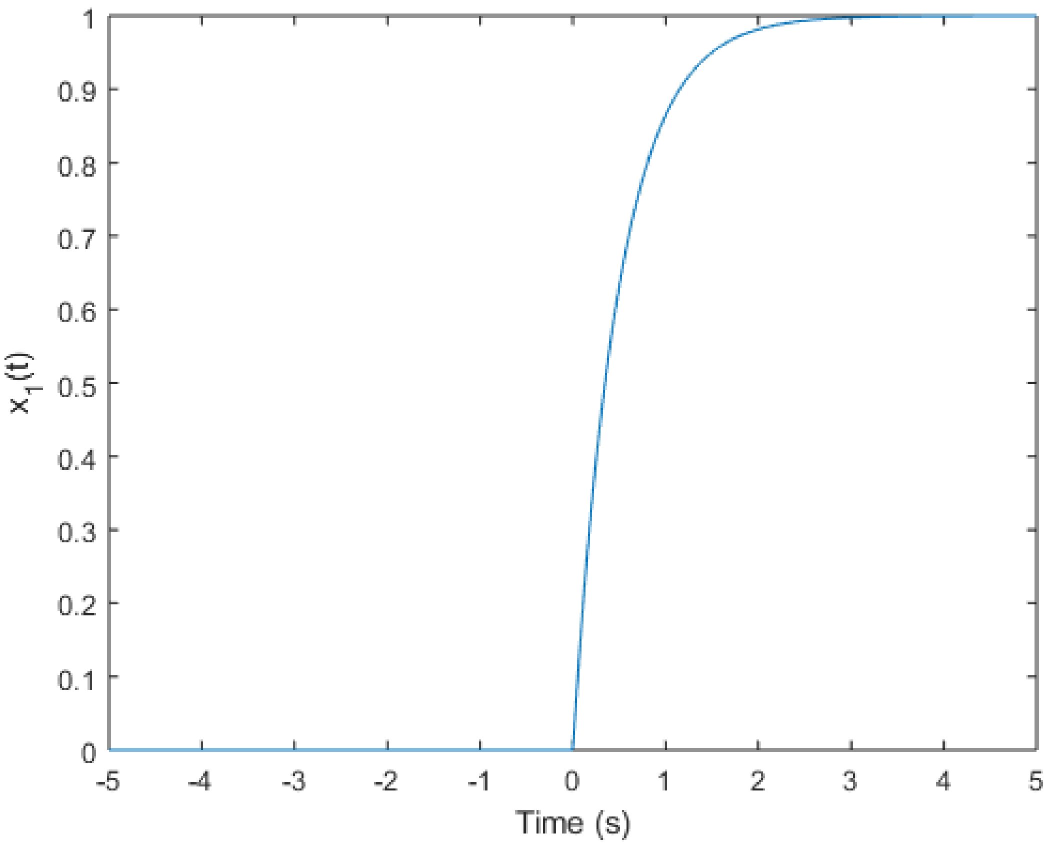
EE3032: Dr. Durant's solution to Problem 1.35, 10/7/2019

Determine if each of the following signals is a power signal, an energy signal, or neither.

a. $x_1(t) = [1 - e^{-2t}]u(t)$

This function is a causal exponential decay towards the DC value of 1. Therefore it has infinite energy but is a power signal. As we consider an infinite time window about t=0, in the limit, the signal is 0 for half of the time and 1 for half of the time; the exponential decay region is vanishingly small around 0+. So, we expect the power to be $(0^2+1^2)/2 = 1/2$.

```
T = 10; % goes to infinity in power definition
t = linspace(-T/2, T/2, 1000);
x1 = (1-exp(-2*t)) .* (t>0);
plot(t, x1)
xlabel('Time (s)')
ylabel('x_1(t)')
```



```
dt = diff(t(1:2));
P = (1/T) * sum(dt*x1.^2); % Riemann integral
fprintf('Using %g samples, the power estimate is %g.\n', ...
    length(t), P)
```

```
Using 1000 samples, the power estimate is 0.425505.
```

The estimate is a bit low since the transient after t=0 is included, but power is a long term average. Increase T for a better estimate.