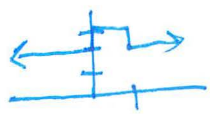
(2.2) (a) $y(t) = \int_{t-1}^{t} x(\tau)\, d\tau + 2$

$x_1(t) = \delta(t)$

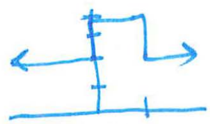
$y_1(t) = \int_{t-1}^{t} \delta(\tau)\, d\tau + 2 = \begin{cases} 2, & t<0 \\ 3, & 0<t<1 \\ 2, & t>1 \end{cases}$ (δ outside of) δ inside of $(t-1, t)$

$x_2 = 2\delta(t)$, so area in intervals is now 0, 2, 0, then add 2

$y_2(t) = \begin{cases} 2 & t<0 \\ 4 & 0<t<1 \\ 2 & t>1 \end{cases}$



$y_1(t)$

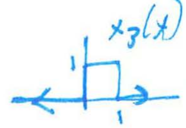


$y_2(t)$

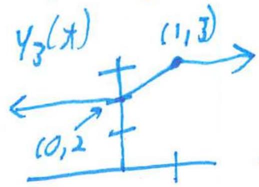
$y_2(t) \neq 2y_1(t)$

$\therefore$ definitely <u>not</u> linear

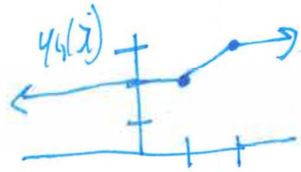
(b) $x_3(t) = u(t) - u(t-1)$



$y_3(t) = 2 +$ area under $x_3(t)$ from $t-1$ to t



$x_4(t) = x_3(t-1) = u(t-1) - u(t-2)$



$y_4(t) = y_3(t-1)$ $\therefore$ system <u>appears</u> time invariant
(It is indeed TI, but this isn't a complete proof.)

(c) No, 0 input $\rightarrow$ output $= 2 \neq 0$

(d) Yes. Let $|x(t)| \leq M$, then $y(t) \leq \left(\int_{t-1}^{t} \overset{\max x(t)}{M}\, d\tau\right) + 2 = M + 2 < \infty$

2.6, (a) only

input $\downarrow$ system function $\downarrow$ bias constant $\swarrow$

$z(t) = v(t) f(t) + B$

(i) $f(t) = A \therefore z(t) = Av(t) + B$

linear system iff $B = 0 \ldots$

"if & only if"

$v(t) = v_1(t) + v_2(t)$

$z_1(t) = Av_1(t) + B \qquad z_2(t) = Av_2(t) + B$

$z_1(t) + z_2(t) = A(v_1(t) + v_2(t)) + 2B$

$z(t) = \qquad A(v_1(t) + v_2(t)) + B$

$(z_1(t) + z_2(t)) - z(t) = 2B - B = B$

$\therefore$ linear iff $B = 0$

(ii) $f(t) = \cos(\Omega_0 t), B = 0, \therefore z(t) = \cos(\Omega_0 t)\, v(t)$

Linear? $v(t) = v_1(t) + v_2(t)$

$z_1(t) = \cos(\Omega_0 t)\, v_1(t) \qquad z_2(t) = \cos(\Omega_0 t)\, v_2(t) \qquad z_1(t) + z_2(t) = \cos(\Omega_0 t)(v_1(t) + v_2(t))$

$z(t) = \cos(\Omega_0 t)\, v(t)$

$z_1(t) + z_2(t) = z(t) \therefore$ linear

TI? Again $z(t) = \cos(\Omega_0 t)\, v(t)$; $z(t-d) = \cos(\Omega_0(t-d))\, v(t-d)$

Now, input $v(t-d)$, $z_d(t) = \cos(\Omega_0 t)\, v(t-d)$

TI iff $z(t-d) = z_d(t) \Rightarrow \cos(\Omega(t-d)) = \cos(\Omega t)$

We can easily pick almost any d to make false, $\therefore$

<u>not</u> TI

(iii) $f(t) = u(t) - u(t-1)$ $\quad v(t) = u(t) - u(t-1)$ $\quad B = 0$

$\therefore z(t) = v(t) f(t) + B = (u(t) - u(t-1))^2 + 0 = u(t) - u(t-1)$

$v_2(t) = v(t-2) = u(t-2) - u(t-3)$

$z_1(t) = v_2(t) f(t) + B = \underbrace{(u(t-2) - u(t-3))}\underbrace{(u(t) - u(t-1))} + 0 = 0 \quad \forall t$

Never overlap

$z_1(t) \neq z(t-2) \therefore$ NOT TI

Expected since gain $f(t)$ is time-varying.

(2.10) (a) $y(t) = |x(t)|^2 = |\cos(t)|^2 = \cos^2(t) = \frac{1}{2}(1 + \cos(2t))$

ω change $\therefore$ not LTI

(b) $y(t) = 0.5(\cos(t) + \cos(t-1)) = 2\cos\left(\frac{2t-1}{2}\right)\cos\left(\frac{1}{2}\right)$

$= \underbrace{2\cos\left(\tfrac{1}{2}\right)}_{\text{gain}} \cos(t \underbrace{-\tfrac{1}{2}}_{\phi \text{ shift}})$

ω unchanged

$\therefore$ LTI

(c) $y(t) = x(t)u(t) = \cos(t)u(t)$

Not a sinusoid since truncated @ 0.

Not LTI.

Specifically, linear, but not TI.

(d) $y(t) = \frac{1}{2}\int_{t-2}^{t} x(\tau)\,d\tau = \frac{1}{2}\int_{t-2}^{t} \cos(\tau)\,d\tau = \frac{1}{2}\sin(t)\Big|_{t-2}^{t}$

$= \frac{1}{2}(\sin(t) - \sin(t-2)) = \cos\left(\frac{2t-2}{2}\right)\sin\left(\frac{2}{2}\right) = \underbrace{\sin(1)}_{\text{gain}} \cos(t \underbrace{-1}_{\phi \text{ shift}})$

ω unchanged

$\therefore$ LTI