Dr. Durant's notes on solutions for fall, 2017 homework. Many of these are not complete solutions, but notes on key points. Feel free to use tools like Wolfram Alpha to check your work. Open up the comments below for more details.

EE-3032, HW-1

Signal properties

1. Consider a triangular pulse, defined as

2pt

$$\Delta(t) \triangleq \begin{cases} 1-2|t| & |t| < \frac{1}{2} \\ 0 & |t| \geq \frac{1}{2} \end{cases}$$

Plot the following signals:

a. $x_1(t) = 6\Delta(t-3)$

b. $x_2(t) = -3\Delta(2t)$ $|t| \leq 1/4$

c. $x_3(t) = \Delta(2(t-3))$ $3 \pm 1/4$

d. $x_4(t) = \Delta(-3t+2)$ $2/3 \pm 1/6$

2. State whether each of the following signals is even, odd, or neither. Justify your answer.

2pt

a. $x_1(t) = rect\left(\frac{t}{T}\right)$ even

b. $x_2(t) = A\cos(\omega_0 t)$ even

c. $x_3(t) = A\sin(\omega_0 t)$ O

d. $x_4(t) = x_1(t)x_2(t)$ E

e. $x_5(t) = x_1(t)x_3(t)$ O

f. $x_6(t) = x_2(t)x_3(t)$ O

g. $x_7(t) = x_1(t) + x_2(t)$ E

h. $x_8(t) = x_2(t) + x_3(t)$ $= A(\cos(\omega_0 t) + \sin(\omega_0 t)) = A\sqrt{2}\sin\left(\omega_0 t + \tan^{-1}\left(\frac{A}{A}\right)\right) = A\sqrt{2}\sin\left(\omega_0 t + \frac{\pi}{4}\right)$

Neither

3. Determine whether each of the following signals is periodic and state the period, if one exists.

2pt

a. $x_1(t) = e^{(-2+j5)t}$ e^{-2t} part → NOT periodic

b. $x_2(t) = e^{j(100\pi t + \pi/6)}$ $\omega = 100\pi$ $F = 50$ $T = \frac{1}{50} = 0.02s$

c. $x_3(t) = \sum_{n=-\infty}^{\infty} \Delta\left(\frac{t-nT_0}{T_1}\right)$ $T = T_0/T_1$

d. $x_4(t) = 5\cos(400\pi t) + 3\sin(500\pi t) + \cos(300\pi t)$

$T_1 = \frac{1}{200}$ $T_2 = \frac{1}{250}$ $T_3 = \frac{1}{150}$ $T = \frac{4}{200} = \frac{5}{250} = \frac{3}{150} = \boxed{\frac{1}{50} = 0.02s}$

4. [Learning objective: power/energy] Calculate the energy or power (as applicable) of the following signals:

2pt

a. $x_1(t) = \Delta\left(\frac{t}{T}\right)$ If $T=1$ $E_{x_1} = 2\int_0^{1/2}(1-2t)^2 dt = 2\int_0^{1/2} 1-4t+4t^2 dt = 2\left[t-2t^2+\frac{4}{3}t^3\right]_0^{1/2} = 2\left(\frac{1}{2}-\frac{1}{2}+\frac{1}{6}\right) = \frac{1}{3}$

b. $x_2(t) = e^{j2\pi t}$ $P_{x_2} = \frac{1}{T}\int_{-T/2}^{T/2} |1|^2 dt = 1$

c. $x_3(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$ $P_{x_3} = \frac{A^2}{2} + \frac{B^2}{2}$

d. $x_4(t) = A\,rect\left(\frac{t}{T}\right)\cos(\omega_0 t)$ $E_{x_4} = A^2\frac{1}{T}\cdot 2\int_0^{T/2}\cos^2(\omega_0 t)\,dt = \frac{2A^2}{T}\int_0^{T/2} \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)\,dt$

$= \frac{2A^2}{T}\left(\frac{T}{4} + \left[\frac{1}{2\omega_0}\frac{1}{2}\sin(2\omega_0 t)\right]_0^{T/2}\right) = \frac{2A^2}{T}\left[\frac{T}{4} + \frac{1}{4}\frac{1}{\omega_0}\sin\left(\frac{\omega_0 T}{2}\right)\right]$

$= \frac{A^2}{2T}\left(T + \frac{1}{\omega_0}\sin\left(\frac{\omega_0 T}{2}\right)\right) = \frac{A^2}{2T}\left(T + \frac{1}{2\omega_0}\sin\left(\frac{\omega_0 T}{2}\right)\right)$

Additional problems from the Chaparro text: 1.3, 1.4

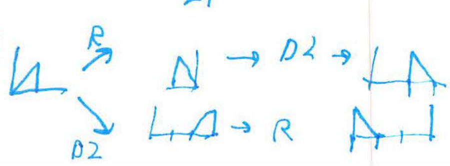
① 1.3 ⓐ Why $e^{-t}u(t)$ causal? ... $t \in \mathbb{R}$, $\alpha > 0$.

ⓑ $x_e = \cos(t)$ $x_o(t) = j\sin(t)$

ⓒ $\int_{-\infty}^{\infty} x_e(t)\sin(\Omega_0 t)\,dt = 0$. Why?

ⓓ Yes. $\int x_e x_o = 0$

① 1.4



≠ ∴ not commute

4a, general case, any $T>0$. Use $\Delta(tT) \neq \Delta(t/T)$ for simplicity

$$E_{x,1} = 2\int_0^{1/(2T)} (1-2tT)^2\, dt = 2\int_0^{\frac{1}{2T}} 1 - 4tT + 4t^2T^2\, dt$$

$$= 2\left[t - 2t^2T + \frac{4}{3}t^3T^2\right]_0^{\frac{1}{2T}}$$

$$= 2\left[\frac{1}{2T} - \frac{1}{2T} + \frac{1}{6T}\right] = \frac{1}{3T}$$

Substituting $v = 1-2tT$ may be easier

Check integrals using an auto-solver, like Wolfram Alpha

This solution is for $\Delta(tT)$, so $\frac{T}{3}$ is result for $\Delta\left(\frac{t}{T}\right)$