Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

EE-3032 – Final Exam – Dr. Durant

November, 2017

May use 8½” × 11” note sheet. No calculator.

Good luck!

Name: ____________________________________

Page 3:	(17 points)	______
Page 4:	(24 points)	______
Page 5:	(16 points)	______
Page 6:	(15 points)	______
Page 7:	(12 points)	______
Page 8:	(16 points)	______
Total:	(100 points)	______

Table 5.1

Basic Properties of Fourier Transform

	Time Domain	Frequency Domain
Signals and constants	$x(t), y(t), z(t), \alpha, \beta$	$X(\Omega), Y(\Omega), Z(\Omega)$
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\Omega) + \beta Y(\Omega)$
Expansion/contraction in time	$x(\alpha t), \alpha \neq 0$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{\Omega}{\alpha}\right)$
Reflection	$x(-t)$	$X(-\Omega)$
Parseval's energy relation	$E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$	$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$
Duality	$X(t)$	$2\pi x(-\Omega)$
Time differentiation	$\frac{d^n x(t)}{dt^n}, n \geq 1$, integer	$(j\Omega)^n X(\Omega)$
Frequency differentiation	$-jtx(t)$	$\frac{dX(\Omega)}{d\Omega}$
Integration	$\int_{-\infty}^{t} x(t')dt'$	$\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$
Time shifting	$x(t - \alpha)$	$e^{-j\alpha\Omega} X(\Omega)$
Frequency shifting	$e^{j\Omega_0 t} x(t)$	$X(\Omega - \Omega_0)$
Modulation	$x(t)\cos(\Omega_c t)$	$0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$
Periodic signals	$x(t) = \sum_k X_k e^{jk\Omega_0 t}$	$X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$
Symmetry	$x(t)$ real	$\lvert X(\Omega)\rvert = \lvert X(-\Omega)\rvert$
		$\angle X(\Omega) = -\angle X(-\Omega)$
Convolution in time	$z(t) = [x * y](t)$	$Z(\Omega) = X(\Omega)Y(\Omega)$
Windowing/Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}[X * Y](\Omega)$
Cosine transform	$x(t)$ even	$X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real
Sine transform	$x(t)$ odd	$X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary

Table 5.2

Fourier Transform Pairs

	Function of Time	Function of Ω
(1)	$\delta(t)$	1
(2)	$\delta(t - \tau)$	$e^{-j\Omega\tau}$
(3)	$u(t)$	$\frac{1}{j\Omega} + \pi\delta(\Omega)$
(4)	$u(-t)$	$\frac{-1}{j\Omega} + \pi\delta(\Omega)$
(5)	$\text{sign}(t) = 2[u(t) - 0.5]$	$\frac{2}{j\Omega}$
(6)	$A, -\infty < t < \infty$	$2\pi A\delta(\Omega)$
(7)	$Ae^{-at}u(t), a > 0$	$\frac{A}{j\Omega + a}$
(8)	$Ate^{-at}u(t), a > 0$	$\frac{A}{(j\Omega + a)^2}$
(9)	$e^{-a\lvert t\rvert}, a > 0$	$\frac{2a}{a^2 + \Omega^2}$
(10)	$\cos(\Omega_0 t), -\infty < t < \infty$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
(11)	$\sin(\Omega_0 t), -\infty < t < \infty$	$-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
(12)	$p(t) = A[u(t + \tau) - u(t - \tau)], \tau > 0$	$2A\tau\frac{\sin(\Omega\tau)}{\Omega\tau}$
(13)	$\frac{\sin(\Omega_0 t)}{\pi t}$	$P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$
(14)	$x(t)\cos(\Omega_0 t)$	$0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$

1. (4 points) ***Sketch*** the ***imaginary*** part of $x(t) = 5e^{(-1+j2\pi)t}\,u(t)$.

2. (4 points) ***Explain*** whether $s(t) = \sin(3\pi t) + \cos(7\pi t)$ is ***periodic***. If it is, calculate its ***fundamental period***.

3. (4 points) ***Explain*** whether $y(t) = \cos(t) + \sin(\pi t)$ is ***periodic***. If it is, calculate its ***fundamental period***.

4. (5 points) ***Calculate $Y(\Omega)$***, the Fourier transform of y(t), or explain why it cannot be done.

5. (7 points) ***Find a*** > 2 such that z(t) = r(t) + r(t-1) – 3r(t-2) + r(t-a) has ***finite energy***. ***Sketch*** the resulting z(t).

6. (6 points) Let w(t) = 4 sin (Ω_1t). ***Fold*** the signal, ***double*** its frequency, ***and then delay*** the result by 1 second.

7. (6 points) Let v(t) = e^{at}u(-t), a>0. ***Decompose v(t)*** into even and odd signals such that v(t) = v_e(t) + v_o(t).

8. (5 points) ***Calculate*** the ***energy or power*** as appropriate of q(t) = (2+3j)$e^{j\pi t/2}$(u(t)-u(t-10)).

9. (6 points) Let x(t) be an unknown system input and y(t) be the corresponding system output. Specifically, let $y(t) = e^{-a|t|}x(t)$, a>0. ***Prove or convincingly explain*** whether this system has each of the following properties:

   a. Linear

   b. Time-invariant

   c. BIBO stable

10. (6 points) Now, consider the system y(t) = x(t)x(t-1). ***Prove or convincingly explain*** whether this system has each of the following properties:

   a. Linear

   b. Time-invariant

   c. BIBO stable

11. (4 points) Which of the following properties are necessary for a system to have an impulse response?

   a. Causal

   b. Linear

   c. BIBO stable

   d. Time-invariant

12. (15 points) Let a system have impulse response $h(t) = e^{-t}u(t)$. Let the system input be $x(t) = u(t) - u(t-3)$. ***Find*** the ***system output*** y(t) ***using convolution***. Hint: there are ***2 non-trivial pieces***. ***Sketch*** your result.

13. (12 points) A periodic signal has the Fourier Series $\{2/5, 0, -2/3, 0, 2, \underline{0}, 2, 0, -2/3, 0, 2/5\}$. $\Omega_0 = 2\pi$.

   a. What is the ***DC offset*** of the signal?

   b. Is the signal ***even, odd, or neither***?

   c. ***Sketch*** the ***power spectrum*** of the signal.

   d. What is the ***power*** of the signal?

   e. What is the Fourier ***transform*** of this periodic signal?

   f. What is the ***time-domain signal*** itself?

14. (4 points) Sketch the ***magnitude response*** from -10 to 10 Hz of a ***bandpass filter*** that passes signals between 2 and 4 Hz, but blocks signals outside of this range.

15. (6 points) Use lines 13 and 14 from Table 5.2 to determine the ***impulse response h(t)*** of this system, assuming that the ***phase shift is 0***.

16. (6 points) Let a system have impulse response $h(t) = 20e^{-4t}u(t)$. Let the system input be the steady state signal $x(t) = \sin(3t)$. What is the ***steady state output***?