Milwaukee School of Engineering

Electrical Engineering and Computer Science Department

# EE-3032 – Final Exam – Dr. Durant

November, 2017

May use 8½” × 11” note sheet. No calculator.

***Good luck!***

Answers 4

Name: Grading Notes

| | | |
|---|---|---|
| Page 3: | (17 points) | ______ |
| Page 4: | (24 points) | ______ |
| Page 5: | (16 points) | ______ |
| Page 6: | (15 points) | ______ |
| Page 7: | (12 points) | ______ |
| Page 8: | (16 points) | ______ |
| Total: | (100 points) | ______ |

**Table 5.1**

**Basic Properties of Fourier Transform**

| | Time Domain | Frequency Domain |
|---|---|---|
| Signals and constants | $x(t), y(t), z(t), \alpha, \beta$ | $X(\Omega), Y(\Omega), Z(\Omega)$ |
| Linearity | $\alpha x(t) + \beta y(t)$ | $\alpha X(\Omega) + \beta Y(\Omega)$ |
| Expansion/contraction in time | $x(\alpha t), \alpha \neq 0$ | $\frac{1}{\lvert\alpha\rvert} X\left(\frac{\Omega}{\alpha}\right)$ |
| Reflection | $x(-t)$ | $X(-\Omega)$ |
| Parseval's energy relation | $E_x = \int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt$ | $E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\Omega)\rvert^2 d\Omega$ |
| Duality | $X(t)$ | $2\pi x(-\Omega)$ |
| Time differentiation | $\frac{d^n x(t)}{dt^n}, n \geq 1$, integer | $(j\Omega)^n X(\Omega)$ |
| Frequency differentiation | $-jtx(t)$ | $\frac{dX(\Omega)}{d\Omega}$ |
| Integration | $\int_{-\infty}^{t} x(t')dt'$ | $\frac{X(\Omega)}{j\Omega} + \pi X(0)\delta(\Omega)$ |
| Time shifting | $x(t-\alpha)$ | $e^{-j\alpha\Omega}X(\Omega)$ |
| Frequency shifting | $e^{j\Omega_0 t}x(t)$ | $X(\Omega - \Omega_0)$ |
| Modulation | $x(t)\cos(\Omega_c t)$ | $0.5[X(\Omega - \Omega_c) + X(\Omega + \Omega_c)]$ |
| Periodic signals | $x(t) = \sum_k X_k e^{jk\Omega_0 t}$ | $X(\Omega) = \sum_k 2\pi X_k \delta(\Omega - k\Omega_0)$ |
| Symmetry | $x(t)$ real | $\lvert X(\Omega)\rvert = \lvert X(-\Omega)\rvert$ |
| | | $\angle X(\Omega) = -\angle X(-\Omega)$ |
| Convolution in time | $z(t) = [x * y](t)$ | $Z(\Omega) = X(\Omega)Y(\Omega)$ |
| Windowing/Multiplication | $x(t)y(t)$ | $\frac{1}{2\pi}[X * Y](\Omega)$ |
| Cosine transform | $x(t)$ even | $X(\Omega) = \int_{-\infty}^{\infty} x(t)\cos(\Omega t)dt$, real |
| Sine transform | $x(t)$ odd | $X(\Omega) = -j\int_{-\infty}^{\infty} x(t)\sin(\Omega t)dt$, imaginary |

**Table 5.2**

**Fourier Transform Pairs**

| | Function of Time | Function of $\Omega$ |
|---|---|---|
| (1) | $\delta(t)$ | 1 |
| (2) | $\delta(t-\tau)$ | $e^{-j\Omega\tau}$ |
| (3) | $u(t)$ | $\frac{1}{j\Omega} + \pi\delta(\Omega)$ |
| (4) | $u(-t)$ | $\frac{-1}{j\Omega} + \pi\delta(\Omega)$ |
| (5) | $\text{sign}(t) = 2[u(t) - 0.5]$ | $\frac{2}{j\Omega}$ |
| (6) | $A, -\infty < t < \infty$ | $2\pi A\delta(\Omega)$ |
| (7) | $Ae^{-at}u(t), a > 0$ | $\frac{A}{j\Omega + a}$ |
| (8) | $Ate^{-at}u(t), a > 0$ | $\frac{A}{(j\Omega + a)^2}$ |
| (9) | $e^{-a\lvert t\rvert}, a > 0$ | $\frac{2a}{a^2 + \Omega^2}$ |
| (10) | $\cos(\Omega_0 t), -\infty < t < \infty$ | $\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$ |
| (11) | $\sin(\Omega_0 t), -\infty < t < \infty$ | $-j\pi[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$ |
| (12) | $p(t) = A[u(t+\tau) - u(t-\tau)], \tau > 0$ | $2A\tau\frac{\sin(\Omega\tau)}{\Omega\tau}$ |
| (13) | $\frac{\sin(\Omega_0 t)}{\pi t}$ | $P(\Omega) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$ |
| (14) | $x(t)\cos(\Omega_0 t)$ | $0.5[X(\Omega - \Omega_0) + X(\Omega + \Omega_0)]$ |

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(-) d(t) (-1) decay
(-) ϕ/cos

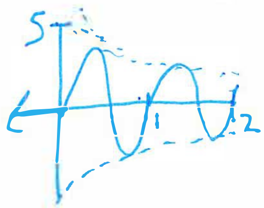
1. (4 points) ***Sketch*** the ***imaginary*** part of $x(t) = 5e^{(-1+j2\pi)t}\,u(t)$.

2. (4 points) ***Explain*** whether $s(t) = \sin(3\pi t) + \cos(7\pi t)$ is ***periodic***. If it is, calculate its ***fundamental period***. (-2) $LCM(\Omega_1, \Omega_2)$

3. (4 points) ***Explain*** whether $y(t) = \cos(t) + \sin(\pi t)$ is ***periodic***. If it is, calculate its ***fundamental period***.

4. (5 points) ***Calculate Y(Ω)***, the Fourier transform of y(t), or explain why it cannot be done.

(-2) saying can't be done since not periodic

(1) 


(2) Yes... $\Omega_1 = 3\pi$ $\Omega_2 = 7\pi$

$f_1 = 3/2$ $f_2 = 7/2$

$T_1 = 2/3$ $T_2 = 2/7$

$T_0 = LCM(T_1, T_2) = 2\ s$ $\left(\frac{1}{2}\ Hz,\ \pi\ \frac{rad}{s}\right)$

(3) No, ratio of freq. is not rational, $1:\pi$.

(4) Use Tables...

$$Y(\Omega) = \pi\left(\delta(\Omega-1) + \delta(\Omega+1) - j\,\delta(\Omega-\pi) + j\,\delta(\Omega+\pi)\right)$$

Common mistake: True that F.S. does not exist. But F.T. does not depend on a fund. period. So, just apply linearity of the F.T. to the 2 components.

(24)

5. (7 points) ***Find a*** > 2 such that $z(t) = r(t) + r(t-1) - 3r(t-2) + r(t-a)$ has ***finite energy***. ***Sketch*** the resulting z(t). (-2) insert δ w/ minor/assoc. error

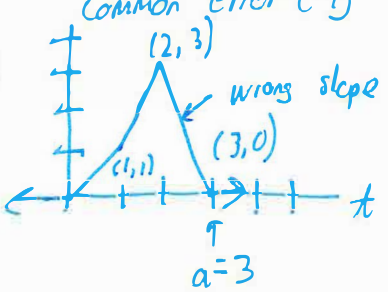
6. (6 points) Let $w(t) = 4\sin(\Omega_1 t)$. ***Fold*** the signal, ***double*** its frequency, ***and then delay*** the result by 1 second. (-1) precedence: t-1 First

7. (6 points) Let $v(t) = e^{at}u(-t)$, $a>0$. ***Decompose v(t)*** into even and odd signals such that $v(t) = v_e(t) + v_o(t)$. (-1) Euler doesn't work, u(t) not E or O

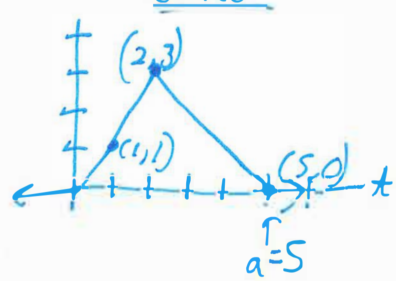
8. (5 points) ***Calculate*** the ***energy or power*** as appropriate of $q(t) = (2+3j)e^{j\pi t/2}(\cancel{u(t-10)-u(t)})$. $u(t)-u(t-10)$

(-1) conj/mag on const (-1) $e^{jx}e^{-jx}=e^0$

Common Error (-1)




Correct




(5)

(6) Fold: $w(-t) = 4\sin(-\Omega_1 t) = -4\sin(\Omega_1 t)$

Double freq: $\rightarrow 4\sin(-2\Omega_1 t) = -4\sin(2\Omega_1 t)$

Delay: $4\sin(-2\Omega_1(t-1)) = -4\sin(2\Omega_1(t-1))$

OR, equiv.:

$4\sin(2\Omega_1(-t+1))$

(7) $v_e(t) = \dfrac{v(t)+v(-t)}{2} = \ldots \dfrac{e^{-|t|a}}{2} = \frac{1}{2}\left(e^{at}u(-t) + e^{-at}u(t)\right)$ simplified basic answer

$v_o(t) = \dfrac{v(t)-v(-t)}{2} = \frac{1}{2}\left(e^{at}u(-t) - e^{-at}u(t)\right) = \dfrac{e^{-|t|a}}{2} \cdot -1 \cdot \mathrm{sgn}(t)$

(8) $E_x = \int_0^{10} q(t)\,q^*(t)\,dt = (2+3j)(2-3j)\int_0^{10} e^{j\frac{\pi t}{2}} e^{-j\frac{\pi t}{2}}\,dt$

$= (4+9)\int_0^{10} e^0\,dt$

$= 130$

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9. (6 points) Let x(t) be an unknown system input and y(t) be the corresponding system output. Specifically, let $y(t) = e^{-a|t|}x(t)$, a>0. ***Prove or convincingly explain*** whether this system has each of the following properties:

   a. Linear yes, no cross effects for $x(t)=x_1(t)+x_2(t)$

   b. Time-invariant no, gain of $e^{-a|t|}$ changes w/ time

   c. BIBO stable yes. Max gain = $\lim_{t\to 0} e^{-a|t|} = 1$

10. (6 points) Now, consider the system y(t) = x(t)x(t-1). ***Prove or convincingly explain*** whether this system has each of the following properties:

    a. Linear no, fails scaling. 2x input → 4x output

    b. Time-invariant yes. (both components delayed together)

    c. BIBO stable yes. M bound input yields $M^2$ bound output.

11. (4 points) Which of the following properties are necessary for a system to have an impulse response?

    a. ~~Causal~~

    b. Linear

    c. ~~BIBO stable~~

    d. Time-invariant

(15)

12. (15 points) Let a system have impulse response $h(t) = e^{-t}u(t)$. Let the system input be $x(t) = u(t) - u(t-3)$. ***Find*** the ***system output*** $y(t)$ ***using convolution***. Hint: there are ***2 non-trivial pieces***. ***Sketch*** your result.

$$y(t) = \begin{cases} 0, & t<0 \\ \int_0^t e^{-\tau}\,d\tau, & 0<t<3 \quad (A) \\ \int_{t-3}^{t} e^{-\tau}\,d\tau, & t>3 \quad (B) \end{cases}$$

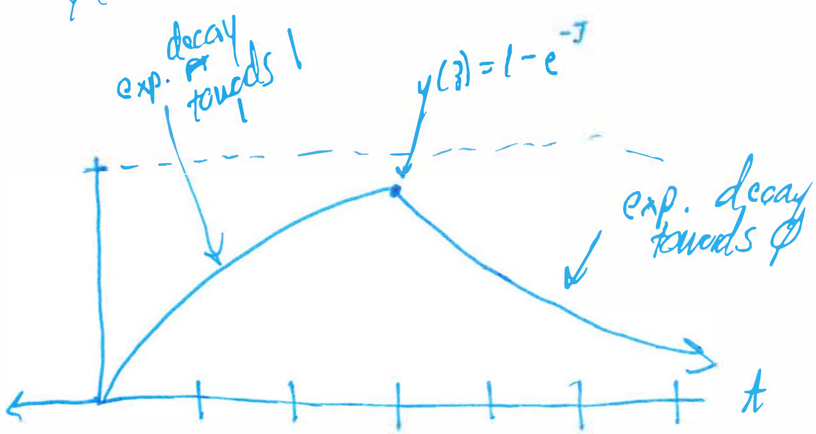
(A) $\int_0^t e^{-\tau}\,d\tau = -1\cdot\left[e^{-\tau}\right]_{\tau=0}^{\tau=t} = -1\left(e^{-t}-1\right) = 1-e^{-t}$

(B) $\int_{t-3}^{t} e^{-\tau}\,d\tau = -1\left[e^{-\tau}\right]_{\tau=t-3}^{\tau=t} = -1\left(e^{-t}-e^{-t+3}\right) = e^{-t+3}-e^{-t}$

Continuity:

$y(3^-) = 1-e^{-3}$

$y(3^+) = e^{0}-e^{-3} = 1-e^{-3} = y(3^-)$ ✓



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$k=-5 \quad -3 \quad -1 \quad 1 \quad 3 \quad 5$

13. (12 points) A periodic signal has the Fourier Series {2/5, 0, -2/3, 0, 2, <u>0</u>, 2, 0, -2/3, 0, 2/5}. $\Omega_0 = 2\pi$.

a. What is the ***DC offset*** of the signal? $X_0 = 0.$

b. Is the signal ***even, odd, or neither***? Even. $X_k \in$ Real $\therefore$ only cosine/even components

c. What is the ***power*** of the signal? $9\frac{47}{225}$, see below

d. What is the Fourier ***transform*** of this periodic signal?

e. ***Sketch*** the ***power spectrum*** of the signal.

f. What is the ***time-domain signal*** itself?

(c) Parseval, $P_x = \sum_k |X_k|^2 = 2(\frac{2}{5})^2 + 2(\frac{+2}{3})^2 + 2(2)^2 =$

$= 2\left(\frac{4}{25} + \frac{4}{9} + 4\right)$

$= 2\left(\frac{36+100}{225} + 4\right)$

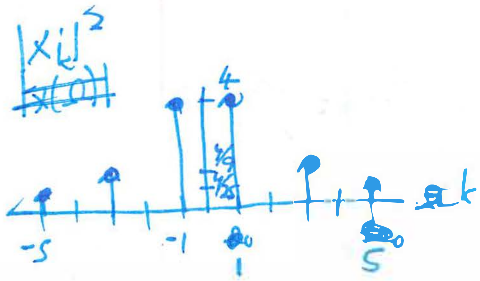
$= 2\left(4\frac{136}{225}\right)$

$= 8\frac{272}{225}$

$= \boxed{9\frac{47}{225}}$

(d) $X(\Omega) = \pi\left[\frac{2}{5}\delta(\Omega + 5\Omega_0) - \frac{2}{3}\delta(\Omega + 3\Omega_0) + 2\delta(\Omega + \Omega_0) + 2\delta(\Omega - \Omega_0) - \frac{2}{3}\delta(\Omega - 3\Omega_0) + \frac{2}{5}\delta(\Omega - 5\Omega_0)\right]$

(e)




~~[illegible]~~
power spectrum $= |X_k|^2$, not $|X(\Omega)|^2$

(f) $x(t) = \frac{4}{5}\cos(5\Omega_0 t) - \frac{4}{3}\cos(3\Omega_0 t) + 4\cos(\Omega_0 t)$

can sub in $\Omega_0 = 2\pi$ & simplify.

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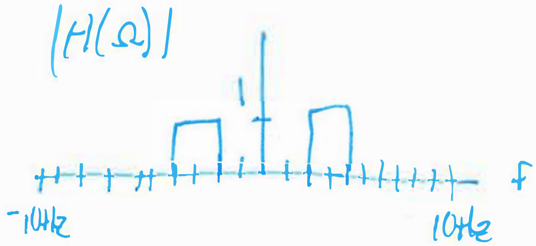
14. (4 points) Sketch the ***magnitude response*** from -10 to 10 Hz of a ***bandpass filter*** that passes signals between 2 and 4 Hz, but blocks signals outside of this range. (-1) neg freq. $\phi$

15. (6 points) Use lines 13 and 14 from Table 5.2 to determine the ***impulse response h(t)*** of this system, assuming that the ***phase shift is 0***.

16. (6 points) Let a system have impulse response $h(t) = 20e^{-4t}u(t)$. Let the system input be the steady state signal $x(t) = \sin(3t)$. What is the ***steady state output***?

(14)




(15) If no $\phi$ shift, (13) gives $h_0(t)$ for the LPF with $|\Omega| < 1\,Hz = \Omega_0$, giving total width of 2Hz

$$h_0(t) = \frac{\sin(2\pi \cdot t)}{\pi t} = \cancel{\quad} = \frac{\sin(2\pi t)}{2\pi t} \cdot \frac{2}{1} = 2\,\mathrm{sinc}(2\pi t)$$

Now, shift right by $3\,Hz = 6\pi \frac{rad}{s}$ so $-1\,Hz < f < 1\,Hz$ becomes $2\,Hz < f < 4\,Hz$

$$h(t) = h_0(t)\cos(6\pi t) = 2\cos(6\pi t)\,\mathrm{sinc}(2\pi t)$$

↑ line (14)

(16) Table 5.2, line 7. $H(\Omega) = \frac{A}{j\Omega + a} = \frac{20}{j\Omega + 4}$

$x(t) = \sin(3t)$, $\therefore \Omega_1 = 3\frac{rad}{s}$

$$H(\Omega_1) = H(3) = \frac{20}{3j+4} \cdot \frac{-3j+4}{-3j+4} = \frac{80 - j60}{16+9} = \frac{16 - j12}{5} = 3.2 - j2.4$$

$$= 4\angle \tan^{-1}\left(\frac{-3}{4}\right)$$

$256 + 144 = 400$

$$|H(3)| = \frac{\sqrt{16^2 + 12^2}}{5} = \frac{\sqrt{400}}{5} = 4$$

$$y(t) = 4\sin\Big(3t + \underbrace{\tan^{-1}\left(\tfrac{-3}{4}\right)}_{\angle H \text{ is } \phi \text{ shift}}\Big) \overset{\tan^{-1}\text{'s odd}}{=} 4\sin\left(3t - \tan^{-1}\left(\tfrac{3}{4}\right)\right)$$

↑ $|H|$ is gain