## Projections

- Mapping 3-D to 2-D
- Parallel projection
  - Preserves relative proportions
  - Unrealistic appearance
- Perspective projection
  - Realistic view
  - Does not preserve proportions

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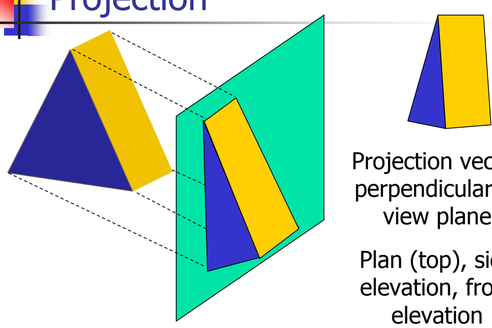
## Orthographic Parallel Projection



Projection vector perpendicular to view plane

Plan (top), side elevation, front elevation

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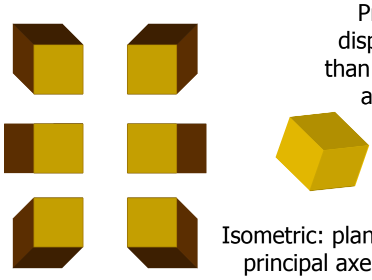
## Axonometric Projections



Projection displays more than one face of an object

Isometric: plane intersects principal axes at same distance

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## Orthographic Transform

- Transform world to view coordinates
  - So direction vector aligned to +z
- Keep x and y coordinates
- Flatten z coordinate
  - Set to zero?
  - Use for depth cueing?

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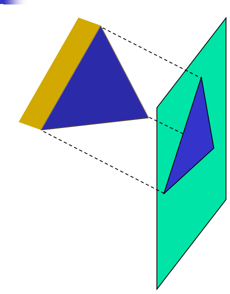
## Oblique Parallel Projection



Projection vector not perpendicular to view plane

"Looking out of the corner of your eye"

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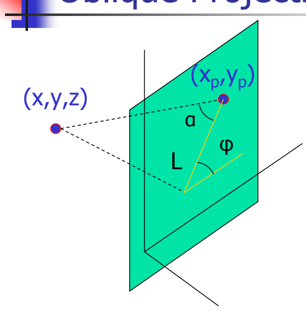
## Oblique Projection Detail




$$x_p = x + L\cos\phi$$

$$y_p = y + L\sin\phi$$

$$\tan\alpha = \frac{z}{L}$$

$$L = \frac{z}{\tan\alpha} = zL_1$$

$$L_1 = \frac{1}{\tan\alpha}$$

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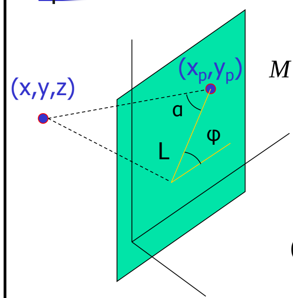
## Parallel Transformation




$$M_{parallel} = \begin{bmatrix} 1 & 0 & L_1 \cos\phi & 0 \\ 0 & 1 & L_1 \sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If $\alpha = 90^o$, then $L_1 = 0$
(orthographic projection)

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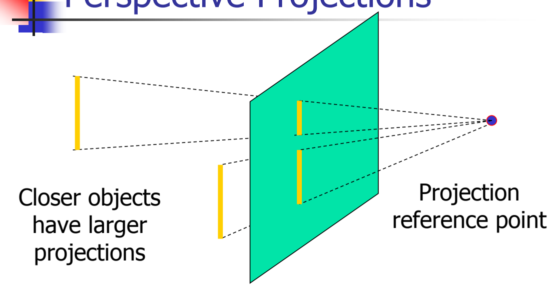
What if view plane beyond reference point?

## Perspective Projections




Parallel lines converge as they move away from plane

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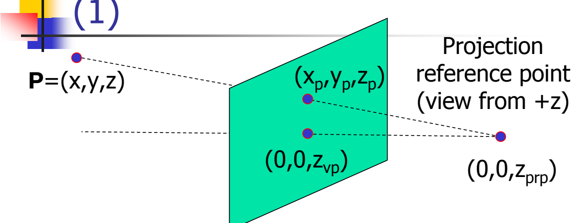
## Perspective Projection Details (1)




Parametric equations for projection line ($0 \le u \le 1$):

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - \left(z - z_{prp}\right)u$$

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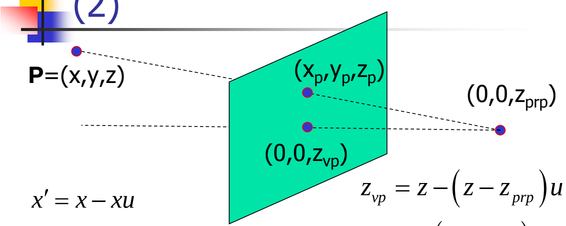
## Perspective Projection Details (2)




$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - \left(z - z_{prp}\right)u$$

$$z_{vp} = z - \left(z - z_{prp}\right)u$$
$$z_{vp} - z = -\left(z - z_{prp}\right)u$$
$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

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## Perspective Projection Details (3)

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - \left(z - z_{prp}\right)u$$

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

$$x' = x - x\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$
$$y' = y - y\left(\frac{z_{vp} - z}{z_{prp} - z}\right)$$
$$z' = z - \left(z - z_{prp}\right)\left(\frac{z_{vp} - z}{z_{prp} - z}\right) = z_{vp}$$

**z in denominator??**

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## Homogeneous Coords. Revisited

- Homogeneous point
  - $P_h = (x_h, y_h, z_h, h)$
- Actual point
  - $P = (x_h/h, y_h/h, z_h/h)$
- So far we've ignored h
  - Set h=1, so $x_h = x$, etc.
  - But h is in denominator! Use it?

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## Perspective Projection Transform

**Choose:** $h = \dfrac{z_{prp} - z}{z_{prp} - z_{vp}} = \dfrac{z_{prp} - z}{d_p}$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-z_{vp}}{d_p} & \frac{z_{vp} z_{prp}}{d_p} \\ 0 & 0 & \frac{-1}{d_p} & \frac{z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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## Check Results — h

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-z_{vp}}{d_p} & \frac{z_{vp} z_{prp}}{d_p} \\ 0 & 0 & \frac{-1}{d_p} & \frac{z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$h = \begin{bmatrix} 0 \\ 0 \\ \frac{-1}{d_p} \\ \frac{z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \frac{-z}{d_p} + \frac{z_{prp}}{d_p} = \frac{z_{prp} - z}{d_p}$$

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## Check Results — x, y

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-z_{vp}}{d_p} & \frac{z_{vp} z_{prp}}{d_p} \\ 0 & 0 & \frac{-1}{d_p} & \frac{z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad x_h = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = x$$

$$x' = \frac{x_h}{h} = \frac{x}{\left(z_{prp} - z\right)/d_p} = \frac{x d_p}{z_{prp} - z} = \frac{x\left(z_{prp} - z_{vp}\right)}{z_{prp} - z}$$

$$= \frac{x\left(z_{prp} - z\right) - x\left(z_{vp} - z\right)}{z_{prp} - z} = x - xu$$

**Same for y**

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## Check Results — z (1)

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-z_{vp}}{d_p} & \frac{z_{vp} z_{prp}}{d_p} \\ 0 & 0 & \frac{-1}{d_p} & \frac{z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$z_h = \begin{bmatrix} 0 \\ 0 \\ \frac{-z_{vp}}{d_p} \\ \frac{z_{vp} z_{prp}}{d_p} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \frac{-z_{vp} z}{d_p} + \frac{z_{vp} z_{prp}}{d_p}$$

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## Check Results — z (2)

$$z_h = \frac{-z_{vp} z}{d_p} + \frac{z_{vp} z_{prp}}{d_p}$$

$$z' = \frac{z_h}{h} = \frac{\left( \frac{-z_{vp} z}{d_p} + \frac{z_{vp} z_{prp}}{d_p} \right)}{\frac{z_{prp} - z}{d_p}} = \frac{z_{vp} z_{prp} - z_{vp} z}{z_{prp} - z}$$

$$= \frac{z_{vp} \left( z_{prp} - z \right)}{z_{prp} - z} = z_{vp}$$

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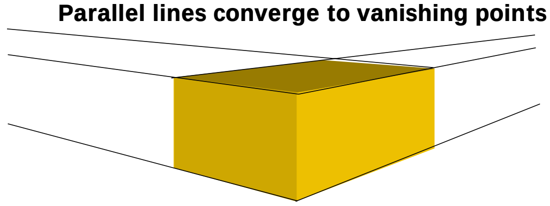
## Vanishing Points

**Parallel lines converge to vanishing points**



**Principal vanishing points correspond to principal axes of object that intersect view plane**

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