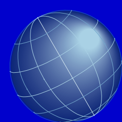
Curved Lines and Surfaces

- Can approximate with polygon mesh
- Also draw curves directly
  - Quadrics
  - Superquadrics
  - Splines
    - Many variations



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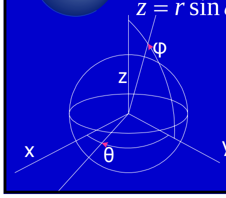
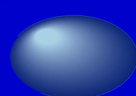
Simple Quadric Surfaces

Sphere

$$x^2 + y^2 + z^2 = r^2$$
$$x = r\cos\phi\cos\theta$$
$$y = r\cos\phi\sin\theta$$
$$z = r\sin\phi$$



Ellipsoid



$$\left(\frac{x}{r_x}\right)^2 + \left(\frac{y}{r_y}\right)^2 + \left(\frac{z}{r_z}\right)^2 = 1$$
$$x = r_x\cos\phi\cos\theta$$
$$y = r_y\cos\phi\sin\theta$$
$$z = r_z\sin\phi$$

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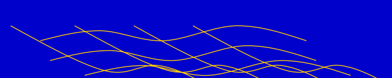
Splines

- Spline curve
  - Composite of polynomial sections
  - Specified continuity conditions
- Spline surface
  - Two sets of orthogonal spline curves



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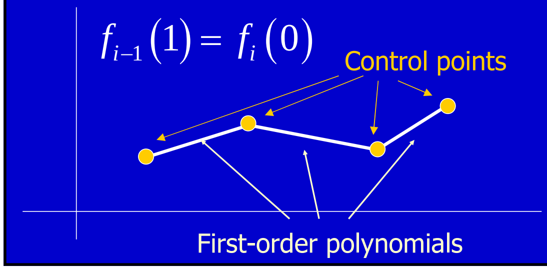
Linear Spline

$$f_i(u) = a_i u + b_i \qquad 0 \le u \le 1$$

$$f_{i-1}(1) = f_i(0)$$



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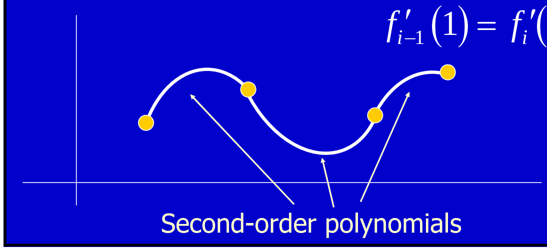
Quadratic Spline

$$f_i(u) = a_i u^2 + b_i u + c_i \qquad f_{i-1}(1) = f_i(0)$$

$$f'_{i-1}(1) = f'_i(0)$$



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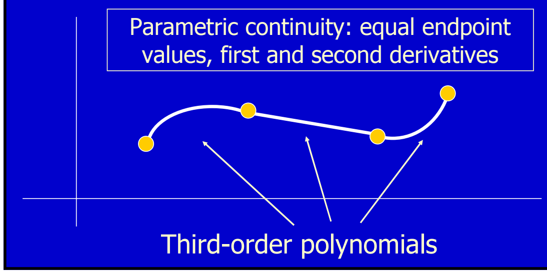
Cubic Spline

$$f_i(u) = a_i u^3 + b_i u^2 + c_i u + d_i$$

Parametric continuity: equal endpoint values, first and second derivatives



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Natural Cubic Spline Calculation

- Solve linear system (Ax=B)
  - Second derivative equations
- Calculate cubic coefficients
- Evaluate function
- Entire spline curve depends on all points

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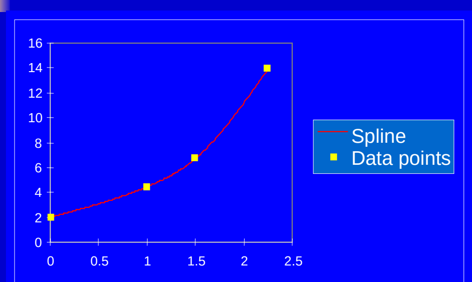
Spline Plot



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Other Splines

- Hermite
  - Each section independent
  - Must specify endpoint derivatives
- Cardinal
  - Adjacent control points determine derivatives
- Bézier, B-splines

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Bézier Splines

- Very popular in graphics
- Control points
  - Determine curve shape
  - Curve need not pass through them
- Polynomial function
  - Degree = # control points - 1

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Bézier Curves

Polynomial: $$\mathbf{P}(u) = \sum_{k=0}^{n} \mathbf{p}_k BEZ_{k,n}(u)$$

n+1 control points: 0..n; u = 0 at p_0, u=1 at p_n

$$BEZ_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$$

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

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Binomial Coefficients

$$C(n,k) = \frac{n!}{k!(n-k)!}$$

n	k: 0	1	2	3	4	5
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5	10	10	5	1

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Bézier Curve: n = 1

$$\mathbf{P}(u) = \sum_{k=0}^{1} \mathbf{p}_k BEZ_{k,1}(u)$$

$$BEZ_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$$

$$BEZ_{0,1}(u) = C(1,0)u^0(1-u)^{1-0} = 1-u$$

$$BEZ_{1,1}(u) = C(1,1)u^1(1-u)^{1-1} = u$$

$$\mathbf{P}(u) = \mathbf{p}_0(1-u) + \mathbf{p}_1 u$$

$$= \mathbf{p}_0 + u(\mathbf{p}_1 - \mathbf{p}_0)$$

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Bézier Curve: n = 2

$$\mathbf{P}(u) = \sum_{k=0}^{2} \mathbf{p}_k BEZ_{k,2}(u)$$

$$BEZ_{k,n}(u) = C(n,k)u^k(1-u)^{n-k}$$

$$BEZ_{0,2}(u) = C(2,0)u^0(1-u)^{2-0} = (1-u)^2$$

$$BEZ_{1,2}(u) = C(2,1)u^1(1-u)^{2-1} = 2u(1-u)$$

$$BEZ_{2,2}(u) = C(2,2)u^2(1-u)^{2-2} = u^2$$

$$\mathbf{P}(u) = \mathbf{p}_0(1-u)^2 + \mathbf{p}_1 2u(1-u) + \mathbf{p}_2 u^2$$

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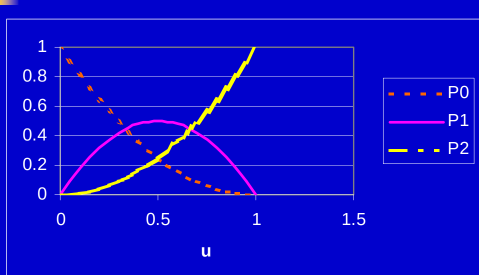
Bézier Curve: n = 2



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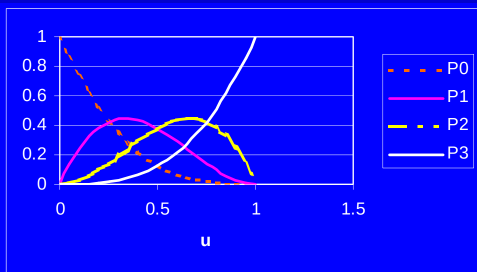
Bézier Curve: n = 3



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Bézier Summary

- Curve shape
  - Influenced by all control points
  - Influence increased near each point
  - Contained in convex hull of control points
- Smooth curve
  - Polynomial degree = #points - 1
  - Usually does not pass through all points

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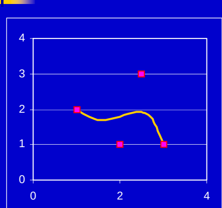
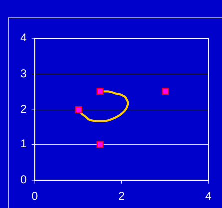
Bézier Curve Examples: n = 3





These are two-dimensional examples;
3-D data works in a similar way.

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