Three-Dimensional Graphics

- True 3-D display (uncommon)
  - "Volume" display technology
    - Moving mirror or target?
    - Stacked display panels?
- Projection to 2-D display
  - Single display
  - Stereoscopic display (two images)

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3-D to 2-D Projection

- Parallel projection
  - Points projected along parallel lines
  - Objects retain original size
  - Parallel lines remain parallel
- Perspective projection
  - Project along converging paths
  - Distant objects appear smaller

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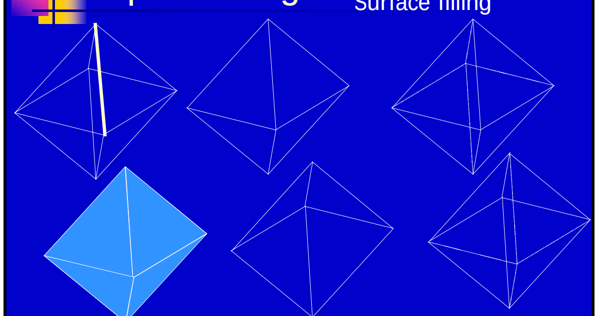
Depth Cueing

Intensity modification
Hidden line removal
Dashed lines
Surface filling



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3-D Object Modeling

- Boundary representations
  - Also known as B-reps
  - Describe set of surfaces
    - That separate object interior from exterior
- Space-partitioning
  - Describe interior as union of solids

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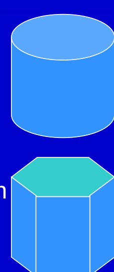
Polygon Surfaces

- Most common B-rep
- All equations linear
- Representing objects
  - Polyhedrons - no problem
  - General shapes - approximation
    - Tessellate to polygon mesh



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Polygon Description

- Geometric data
  - Description of position and shape
- Attribute data
  - Description of surface
    - Transparency
    - Color, surface reflectivity
    - Texture, etc.

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Polygon Geometric Data

- List of vertices (3-D) ?
  - Sufficient description
  - But, polygons are joined
    - Sharing vertices and edges
- General description
  - Reduce redundancy
  - Represent component polygons

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Polygon Tables

- Vertex table
  - For all polygons in composite object
- Edge table
  - Each edge listed once
    - Even if part of more than one polygon
- Polygon-surface table

Pointers or other links between tables for easy access

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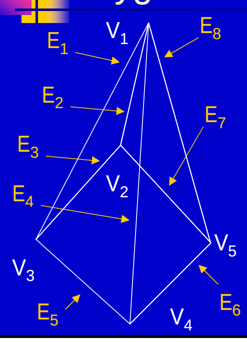
Polygon Table Example



E_1: V_1,V_3
E_2: V_1,V_2
E_3: V_2,V_3
E_4: V_1,V_4
E_5: V_3,V_4
E_6: V_4,V_5
E_7: V_2,V_5
E_8: V_1,V_5

S_1: E_1, E_2, E_3
S_2: E_2, E_7, E_8
S_3: E_1, E_4, E_5
S_4: E_4, E_6, E_8
S_5: E_3, E_5, E_6 , E_7

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Plane Equations

- Processing 3-D object
  - Coordinate transformations
  - Visible surface identification
  - Surface rendering
- Often need information
  - Spatial orientation of surfaces
  - Equations of polygon planes

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Plane Equation

$$Ax + By + Cz + D = 0$$

Must be satisfied for any point (x,y,z) in the plane.

We want to solve for coefficients (A, B, C, D).

However, we know a plane is determined by 3 non-collinear points, so we should only need three equations. But we have 4 unknowns!

Since both sides of the equation can be multiplied by a constant, solve for A/D, etc.

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Plane Equation Solution (1)

$$Ax + By + Cz + D = 0$$

$$\frac{A}{D}x + \frac{B}{D}y + \frac{C}{D}z = -\frac{D}{D} = -1$$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} A/D \\ B/D \\ C/D \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

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Plane Equation Solution (2)

$$A = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2)$$
$$B = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2)$$
$$C = x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$$
$$D = -x_1(y_2z_3 - y_3z_2) - x_2(y_3z_1 - y_1z_3) - x_3(y_1z_2 - y_2z_1)$$

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Vector Formulation (1)

For the plane equation: $Ax + By + Cz + D = 0$

A vector normal to the plane surface is: $\mathbf{N} = (A, B, C)$

Vector **N** is also the cross product of two successive polygon edges. We go CCW around the polygon to find the "inside-to-outside" normal vector: $\mathbf{N} = (\mathbf{V}_2 - \mathbf{V}_1) \times (\mathbf{V}_3 - \mathbf{V}_2)$

But the book says: $\mathbf{N} = (\mathbf{V}_2 - \mathbf{V}_1) \times (\mathbf{V}_3 - \mathbf{V}_1)$

Does it make a difference?

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Vector Formulation (2)

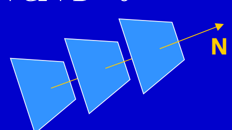
The normal vector:

$$\mathbf{N} = (\mathbf{V}_2 - \mathbf{V}_1) \times (\mathbf{V}_3 - \mathbf{V}_2) = (A, B, C)$$

Gives us values for A, B, and C in the plane equation: $Ax + By + Cz + D = 0$

But what about D?

D specifies which one of the planes normal to **N** we are talking about.



Calculate D by substituting A, B, C, and one polygon vertex into plane equation.

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Vector Formulation (3)

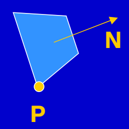
Solving for D in the plane equation:

$$Ax + By + Cz + D = 0$$

$$Ax + By + Cz = -D$$

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -D$$

$$\mathbf{N} \cdot \mathbf{P} = -D$$



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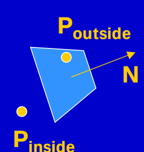
Testing Points

If point is “inside” the plane, then:

$$Ax + By + Cz + D < 0$$

If point is “outside” the plane, then:

$$Ax + By + Cz + D > 0$$



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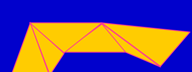
Polygon Meshes

- To approximate general surface
  - Replace it with an approximation
- Set of polygons
  - Triangles
    - Advantage: 3 vertices determine plane
  - Quadrilaterals



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