Line-Drawing Algorithms

- Simple algorithm
  - Evaluate y = f(x) at each x position
  - Floating multiplication
- DDA algorithm
  - Floating addition only
- Can we do better?
  - Bresenham's algorithm

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Who Is Bresenham?

Bresenham, J. E. Algorithm for computer control of a digital plotter, *IBM Systems Journal*, 4(1), 1965, pp. 25-30.

Bresenham, J. E. A linear algorithm for incremental digital display of circular arcs. *Communications of the ACM*, 20(2), 1977, pp. 100-106.

Note the dates!

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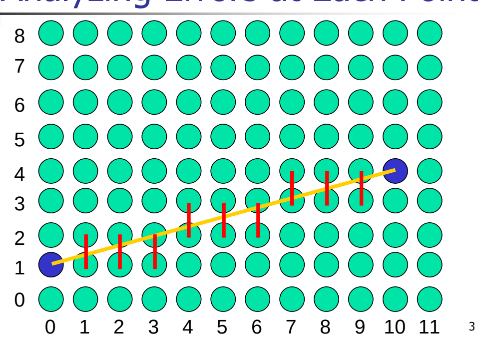
Analyzing Errors at Each Point



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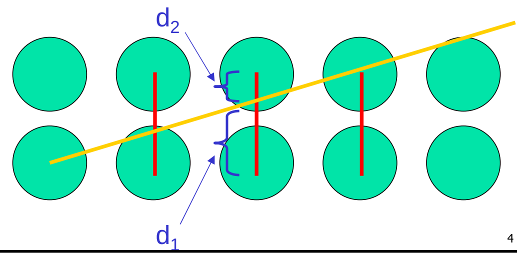
Error Analysis Detail

Represent error for each pixel choice



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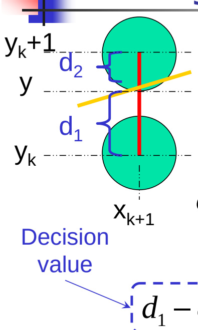
Deciding on the Next Point



$$y = m(x_k + 1) + b$$

$$d_1 = y - y_k$$

$$= m(x_k + 1) + b - y_k$$

$$d_2 = (y_k + 1) - y$$

$$= y_k + 1 - m(x_k + 1) - b$$

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

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Status Check

- What have we accomplished?
  - One value to test (+/–)
- But …
  - Still have floating-point calculation
    - "m" and "b" are non-integer

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

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Removing Non-Integral Values

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2b - 1$$

$$m = \frac{\Delta y}{\Delta x}$$

$$d_1 - d_2 = 2\frac{\Delta y}{\Delta x}(x_k + 1) - 2y_k + 2b - 1$$

$$p_k = \Delta x(d_1 - d_2) = 2\Delta y \cdot x_k + 2\Delta y - 2\Delta x \cdot y_k + \Delta x(2b - 1)$$

$$p_k = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

$$c = 2\Delta y + \Delta x(2b - 1)$$

Decision value (p_k)

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Recurrence Relation for p_k

$$p_k = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x\underbrace{(y_{k+1} - y_k)}_{0 \text{ or } +1}$$

0 or +1

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Initializing the Algorithm

Initial value of decision parameter

$$p_0 = 2\Delta y \cdot x_0 - 2\Delta x \cdot y_0 + c$$

$$p_0 = 2\Delta y \cdot x_0 - 2\Delta x \cdot y_0 + 2\Delta y + \Delta x(2b - 1)$$

But normalize: $x_0 = y_0 = b = 0$

$$p_0 = 2\Delta y - \Delta x$$

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Bresenham Algorithm Summary

Calculate: $\Delta x, \Delta y, 2\Delta y, 2\Delta y - 2\Delta x, p_0$

At each point:

If $p_k < 0$: $\text{Plot}(x_k + 1, y_k)$

$p_{k+1} = p_k + 2\Delta y$

If $p_k >= 0$: $\text{Plot}(x_k + 1, y_k + 1)$

$p_{k+1} = p_k + 2\Delta y - 2\Delta x$

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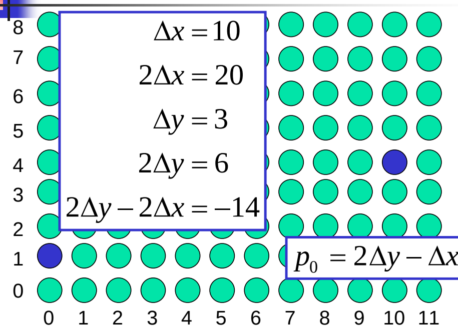
Example 1 Parameters



$\Delta x = 10$

$2\Delta x = 20$

$\Delta y = 3$

$2\Delta y = 6$

$2\Delta y - 2\Delta x = -14$

$p_0 = 2\Delta y - \Delta x = -4$

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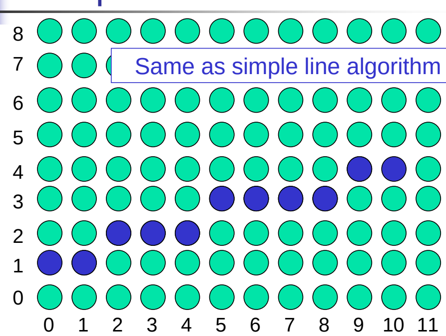
Example 1 Path



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Bresenham Summary

- Only integer arithmetic
- Sign detection
  - Magnitude not tested
- Only multiply is by two
  - Left shift (<<) operator

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Bresenham Modifications

- So far, $0 < m < 1$, $\Delta x > 0$
- How handle other cases?
  - Horizontal
  - Vertical
  - Other octants
    - abs(Slope) $\in (1, \infty]$
      - Transpose x with y, Δx with Δy, etc.
      - Plot(y,x)
    - Slope < 0
      - Revise: $y_{k+1} - y_k = 0$ or -1

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Alternative Viewpoint

- Assume $0 \leq$ slope ≤ 1
- Each step in x moves y up a fraction (Δy/Δx)
- As soon as the fraction exceeds 1/2, move up rather than over (subtract 1 [Δx/Δx] to get back to fraction between –1/2 and 1/2)
- Clear the floats by multiplying by 2Δx in all terms

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