

CE-1901-11 - Dr. Durant - Quiz 7  
Fall 2016, Week 8 Quiz

1. (4 points) Glitches
  - a. **Derive** the minimized SOP expression for  $F(ABCD) = \sum_m(2,5,6,7,8,12,13,15)$ .
  - b. **Determine** between which pair(s) of minterms a (1-bit input change) glitch may occur.
  - c. **Modify** the expression for F to avoid all glitches.
2. (2 points) **Write** the logic equation for the 8:1 MUX,  $y = f(s,d)$ , where s and d are 0-based vectors of appropriate length. Remember that there is a product term for each of the data terms; this product term checks all values of s to make sure they match the term's corresponding s-minterm.

① (a)

	CD	00	01	11	10
AB					
00					1
01			1	1	
11	1	1	1		
10	1				

$$F = BD + \underbrace{\bar{A}\bar{C}\bar{D} + A\bar{C}\bar{D}}_{\text{Min SOP}} = BD + (\bar{A} + A)\bar{C}\bar{D} = BD + (A + \bar{A})\bar{C}\bar{D}$$

(b) (6+7) + (12+13)

(c)  $F = BD + \bar{A}\bar{C}\bar{D} + A\bar{C}\bar{D} + \bar{A}BC + A\bar{B}\bar{C}$

② 8:1  $\rightarrow$  8 data bits  $\rightarrow \lceil \log_2(8) \rceil = 3$  select bits

$\uparrow$   $\uparrow$   
 7..0 2..0  $\in$  0-based

$$y = \bar{s}_2 \bar{s}_1 \bar{s}_0 d_0 + \bar{s}_2 \bar{s}_1 s_0 d_1 + \bar{s}_2 s_1 \bar{s}_0 d_2 + \bar{s}_2 s_1 s_0 d_3 + s_2 \bar{s}_1 \bar{s}_0 d_4 + s_2 \bar{s}_1 s_0 d_5 + s_2 s_1 \bar{s}_0 d_6 + s_2 s_1 s_0 d_7$$

3. (2 points) Recall that the carry-lookahead adder (CLA) uses propagate and generate signals  $p_i = a_i + b_i$  and  $g_i = a_i b_i$ . For a 4-bit CLA, **write** the **2-level** carry out ( $c_4$ ) equation in terms of the propagate and generate signals.
4. (2 points) Assume that gate delay  $\tau = 50$  ps. **Approximate** the speed (propagation delay) of **both** a 64-bit RCA **and** a CLA using 4, **16-bit** lookahead modules. By **what factor** is the CLA faster? Note: make reasonable approximations about the number of gate delays; your answer need not be exact.

$$\textcircled{3} \quad C_4 = g_3 + g_2 p_3 + g_1 p_2 p_3 + g_0 p_1 p_2 p_3 + c_0 p_0 p_1 p_2 p_3$$

$$\textcircled{4} \quad \text{RCA: 2 delays per bit} \rightarrow \boxed{128\tau}$$

$$\text{CLA: bit } p \text{ or } g \text{ @ } \tau = \tau$$

$$\text{block } p \text{ or } g \text{ @ } \tau = 3\tau$$

$$C_{IN} \text{ @ } 16, 32, 48 \text{ @ } 5\tau, 7\tau, 9\tau$$

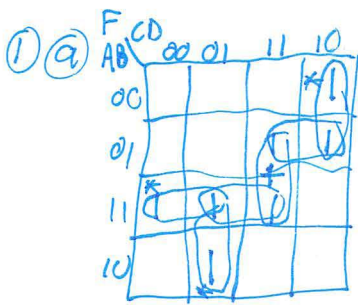
$$\text{RCA } 48 \dots 63 \text{ starts @ } \leftarrow \tau, (9 + 16 \times 2)\tau = \boxed{41\tau}$$

$$\text{factor of speedup} = \frac{128}{41} = \boxed{3.12 \dots}$$

$$41 \overline{) 128} \begin{array}{r} 3.12 \dots \\ \underline{123} \\ 50 \\ \underline{41} \\ 90 \\ \underline{82} \\ 8 \end{array}$$

CE-1901-12 - Dr. Durant - Quiz 7  
Fall 2016, Week 8 Quiz

1. (4 points) Glitches
  - a. **Derive** the minimized SOP expression for  $F(ABCD) = \Sigma_m(2,6,7,9,12,13,15)$ .
  - b. **Determine** between which pair(s) of minterms a (1-bit input change) glitch may occur.
  - c. **Modify** the expression for  $F$  to avoid all glitches.
2. (2 points) **Write** the logic equation for the 8:1 MUX,  $y = f(s,d)$ , where  $s$  and  $d$  are 0-based vectors of appropriate length. Remember that there is a product term for each of the data terms; this product term checks all values of  $s$  to make sure they match the term's corresponding  $s$ -minterm.



\*  $F = \underbrace{AB\bar{C} + A\bar{C}D + \bar{A}C\bar{D}}_{\text{essential}} + \underbrace{BCD}_{\text{simplest way to get 2 final minterms}}$

② (b)  $(6+7) + (13+15)$

③  $F = \underbrace{AB\bar{C} + A\bar{C}D + \bar{A}C\bar{D} + BCD}_{\text{ORIGINAL}} + \underbrace{ABD + \bar{A}BC}_{\text{glitch prevention}}$

②  $8:1 \rightarrow \text{data length} = 8 \rightarrow \text{select length} = \lceil \log_2(8) \rceil = 3$   
 $\uparrow$   $\uparrow$   
 $0..7$   $0..2 \in q\text{-base}$

$$y = \bar{s}_2 \bar{s}_1 \bar{s}_0 d_0 + \bar{s}_2 \bar{s}_1 s_0 d_1 + \bar{s}_2 s_1 \bar{s}_0 d_2 + \bar{s}_2 s_1 s_0 d_3 + s_2 \bar{s}_1 \bar{s}_0 d_4 + s_2 \bar{s}_1 s_0 d_5 + s_2 s_1 \bar{s}_0 d_6 + s_2 s_1 s_0 d_7$$

3. (2 points) Recall that the carry-lookahead adder (CLA) uses propagate and generate signals  $p_i = a_i + b_i$  and  $g_i = a_i b_i$ . For a 4-bit CLA, **write the 2-level** carry out ( $c_4$ ) equation in terms of the propagate and generate signals.
4. (2 points) Assume that gate delay  $\tau = 50$  ps. **Approximate** the speed (propagation delay) of **both** a 64-bit RCA **and** a CLA using 16, **4-bit** lookahead modules. By **what factor** is the CLA faster? Note: make reasonable approximations about the number of gate delays; your answer need not be exact.

③  ~~$c_4 = g_4 + g_3 p_4 + g_2 p_3 p_4 + g_1 p_2 p_3 p_4$~~

$c_4 = g_3 + g_2 p_3 + g_1 p_2 p_3 + g_0 p_1 p_2 p_3$   $+ c_0 p_0 p_1 p_2 p_3$

④ RCA  $\sim 2$  delays per bit,  $64 \times 2\tau = \boxed{128\tau}$

CLA: bit  $p+g @ \tau$

block  $p+g @ 3\tau$   $\xrightarrow{2 \text{ layers}}$   
 Fast carry ins:  $c_4 @ 5\tau, c_8 @ 7\tau$   
 ...  $c_{60} @ 33\tau$

Final RCA4 =  $(33 + (4 \times 2))\tau = \boxed{41\tau}$

Factor =  $\frac{128\tau}{41\tau} = \boxed{3.12...}$

41  $\overline{)128}$   
 127  
 ---  
 50  
 41  
 ---  
 90  
 82  
 ---

$i$	$c_i$	$\tau$
4		5 $\tau$
8		7 $\tau$
12		9 $\tau$
16		11 $\tau$
20		13 $\tau$
24		15 $\tau$
28		17 $\tau$
32		19 $\tau$
36		21 $\tau$
40		23 $\tau$
44		25 $\tau$
48		27 $\tau$
52		29 $\tau$
56		31 $\tau$
60		33 $\tau$

Fast Carry  $@ \left(\frac{i}{4} \cdot 2 + 3\right)\tau$   
 ↑  
 1/4 bits block  
 layers per block